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DISCUSSION

ON

THE ABSOLUTE MEASUREMENT OF ELECTRICAL RESISTANCE, AND INSTRUMENTS BASED ON THE TEMPERATURE-VARIATION OF RESISTANCE.

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I.

THE ABSOLUTE MEASUREMENT OF RESISTANCE.

1. "*Historical Account of the Absolute Determination of the Unit of Electrical Resistance,*" *by* SIR RICHARD GLAZEBROOK, *K.C.B., F.R.S.*

Sir RICHARD GLAZEBROOK gave an historical outline of some of the chief events in the establishment of the C.G.S. unit of electrical resistance, or rather of its multiple, the ohm, and of the standards which represent this; the ground covered being in the main from the initiation in 1861 of the British Association Committee on Electrical Standards to its dissolution in 1912.

At the date first mentioned a great assortment of arbitrary units of electrical resistance were in use, and the question of a definitely recognised standard was a pressing one. Latimer Clark and Sir Charles Bright read a Paper before the British Association, the outcome of which was the appointment, on the motion of Professor William Thomson, of the Committee referred to above. The work of this committee has proved to be of fundamental importance in the history of electrical measurements.

Wilhelm Weber had in 1851 proposed a system of absolute units in regard to electrical quantities. The principle was at once taken up and developed by Thomson, and in 1863, as the result largely of his advocacy, the C.G.S. system of electromagnetic units was adopted by the Committee. As a practical unit of convenient value, the multiple 10^9 units was adopted; for this the term Ohmad was suggested, which later took the abbreviated form Ohm.

The next step was the production of a concrete standard of resistance to represent the Ohm. This involved the determination in absolute measure of the resistance of a particular coil. The method chosen by the Committee was that of the spinning coil, devised by Thomson, though apparently here too Weber had anticipated it several years before. The experiments were carried out at King's College, London, by Clerk Maxwell, Fleeming Jenkin, and Charles Hockin. Two series of experiments were made, one in 1863, the second in 1864. The resistances of two wire coils were thus determined at specified temperatures. On the basis of these, ten standards—eight wire coils and two mercury tubes—were constructed by Matthiessen and Hockin, to be deposited at Kew Observatory, each having a value equal to one ohm at some specified temperature. As a matter of historical interest the data referring to these coils are reproduced in Table I. The initials in the last column denote Matthiessen and Hockin. The Second Report of the Committee in 1863 contains, in addition to a

Table I.—*This Table shows the value of the different copies of the B.A. Units that have been made for preservation at Kew* :—

Material of coil.	No. of coil.	Date of observation.	Temp. at which coil has a resistance 10^9 cm./sec.	Observer.
Platinum-iridium alloy	2	Jan. 4, 1865	15·5°C.	C.H.
		June 6, 1865	16·0°C.	A.M.
		Feb. 10, 1867	16·0°C.	C.H.
Paltinum-iridium alloy	3	Jan. 4, 1865	15·3°C.	C.H.
		June 6, 1865	15·8°C.	A.M.
		Feb. 10, 1867	15·8°C.	C.H.
Gold-silver alloy ...	10	Jan. 5, 1865	15·6°C.	A.M.
		Feb. 10, 1867	15·6°C.	C.H.
Gold-silver alloy ...	58	April 10, 1865	15·3°C.	A.M.
		June 6, 1865	15·3°C.	A.M.
		Feb. 10, 1867	15·3°C.	C.H.
Platinum	35	Jan. 7, 1865	15·7°C.	C.H.
		Aug. 18, 1866	15·7°C.	A.M.
		Feb. 10, 1867	15·7°C.	C.H.
Platinum	36	Jan. 7, 1865	15·5°C.	C.H.
		Aug. 18, 1866	15·5°C.	A.M.
		Feb. 10, 1867	15·7°C.	C.H.
Platinum-silver alloy...	43	Feb. 15, 1865	15·2°C.	C.H.
		Mar. 9, 1865	15·2°C.	A.M.
		Feb. 10, 1867	15·2°C.	C.H.
*Mercury	I.	Feb. 2, 1865	16·0°C.	A.M.
		July 18, 1866	16·0°C.	A.M.
		Feb. 11, 1867	16·7°C.	C.H.
Mercury	II.	Feb. 3, 1865	14·8°C.	A.M.
		Aug. 18, 1866	14·8°C.	A.M.
		Feb. 11, 1867	14·8°C.	C.H.
Mercury	III.	Feb. 11, 1867	17·9°C.	C.H.

* The alteration of this coil, observed on Feb. 11, 1867, is due, no doubt, to a defect observed in the glass tube.

description of the experiments of that year, what amounts to a fundamental treatise on electrical units and measurements.

At that time little was known as to the degree of constancy of the resistance of a coil under various physical conditions or with lapse of time. Valuable data were acquired on these points in the years following. The extent of the secular variation in the case of some of the B.A. standard coils may be seen from Table II.

TABLE II.—*Resistances at* 16·0°*C. in terms of the original B.A. Unit* (1867). (*Values obtained through the two Platinum Coils, D, E.*).

Coil.	Material.	1867	1876	1879-81	1888	1908	Max. diff. B.A.U.
A	Pt Ir	1·00000	1·00077	1·00056	1·00147	1·00122	147×10^{-5}
B	Pt Ir	1·00029	1·00121	1·00080	1·00104	1·00098	92 „
C	Au Ag	1·00050	1·00141	1·00101	1·00146	1·00173	123 „
D	Pt	1·00092	1·00092	1·00092	1·00092	1·00092	0 „
E	Pt	1·00152	1·00152	1·00152	1·00152	1·00152	0 „
F	Pt Ag	...	...	1·00016	1·00072	1·00160	144 „
G	Pt Ag	1·00022	1·00030	0·99982	1·00025	1·00175	193 „
H	Pt Ag	1·00020	...	...	1·00042	1·00044	24 „
Flat	Pt Ag	...	...	1·00079	1·00120	1·00125	46 „

In the meantime other methods of measuring electrical resistance in absolute measure had been proposed, and some results had been obtained—by Kohlrausch and by Rowland, and from the measurements of Joule—which appeared to show an error in the original measurements of the Committee. As a result the Committee, which had not met since 1870, was reappointed in 1880. In 1881 and 1882 several papers* appeared from the hand of Lord Rayleigh, who made a repetition of the experiments with the original spinning coil used by Maxwell, and also a series of experiments with a new apparatus of the same design. For a third series Lord Rayleigh employed the Lorenz method of measurement. The accounts of these, and a critical account of the principal methods that had been used, aroused intense interest. The result was to show that the B.A. ohm was nearly one and a half per cent. too low, probably owing to an underestimate of the value of the self-inductance of the revolving coil.

In 1881 the first International Congress on Electrical Units was held at Paris; this led up to the adoption of the C.G.S. system of units internationally. At this Conference it was agreed that the term ohm should be given to the resistance of a column of mercury 1 mm.^{2} in section at 0°C., and that an international commission should determine the length of this column. Various determinations were made, and in 1884, when the Congress met again, Mascart presented a table showing a mean length of 106·02 cm. The figure 106 cm. was adopted, and defined the "Legal Ohm" of the Paris Congress.

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* "Proc." Roy. Soc., 1881; "Phil. Trans.," 1882; "Phil. Mag.," 1882.

This was, however, felt to be unsatisfactory; the value 106·0 was not that resulting from the best observations, but represented the mean. In 1890 the Board of Trade Committee on Electrical Standards was established; and basing their decision on the results given in Table III., they adopted the length 106·3 cm. This result was confirmed at the Edinburgh meeting of the British Association in 1892, at which Helmholtz, Lindeck, Kahle, Guillaume, and Carhart were present. Helmholtz made the proposal that in place of specifying the sectional area of the column of mercury, the mass of mercury should be substituted; pointing out that in this way the uncertainty in regard to the precise value of the density of mercury would be eliminated. This suggestion was adopted.

Table III.—*Results used in assigning the value of the Ohm. (Edinburgh B.A. Meeting, 1892.)*

No.	Observer.	Date.	Method.	Value of B.A. units in ohms.	Value of ohm in centimetres of mercury.
1	Lord Rayleigh...	1882	Rotating coil...	0·98651	106·31
2	Lord Rayleigh...	1883	Lorenz ...	0·98677	106·27
3	Mascart ...	1884	Induced currents	0·98611	106·33
4	Rowland ...	1887	Mean of several methods	0·98644	106·32
5	Kohlrausch ...	1887	Damping of magnets	0·98660	106·32
6	Glazebrook ...	1882 &1888	Induced currents	0·98665	106·29
7	Wuilleumeier ...	1890	...	0·98686	106·31
8	Duncan&Wilkes	1890	Lorenz ...	0·98634	106·34
9	Viriamu Jones	1891	Lorenz ...	...	106·31
					106·31 mean

The great superiority of the specification of the International Ohm in terms of a column of mercury, rather than in terms of a coil of wire, may be seen from a comparison of the results of Table IV. with those of Table II. The former Table is taken from figures obtained by F. E. Smith, representing the results of a series of fillings in the reproduction, according to specification, of the International Ohm.

TABLE IV.—*Values obtained in a series of mercury fillings in reproduction of the International Ohm (by F. E. Smith).*

Glass.	Length at 0°C. mm.	Calibre factor.	Mass of mercury at 0°C.	Calculated resistance in ohms.	Observed resistance in terms of mean of all.	Difference abs.-calc. 1×10^{-5}
Jena ...	1028·879	$1{\cdot}00010_1$	$13{\cdot}5540_1$	$0{\cdot}99986_6$	$0{\cdot}99986_0$	−0·6
„ ...	1058·125	$1{\cdot}00007_7$	$14{\cdot}3339_3$	$0{\cdot}99994_5$	$0{\cdot}99993_7$	−0·8
„ ...	1000·756	$1{\cdot}00011_6$	$12{\cdot}8274_9$	$0{\cdot}99956_7$	$0{\cdot}99957_9$	+1·2
„ ...	929·586	$1{\cdot}00018_1$	$11{\cdot}0679_7$	$0{\cdot}99965_6$	$0{\cdot}99967_0$	+1·4
„ ...	812·187	$1{\cdot}00007_3$	$8{\cdot}4475_0$	$0{\cdot}99977_1$	$0{\cdot}99976_1$	−1·0
Verre dur	1188·249	$1{\cdot}00002_6$	$18{\cdot}0776_2$	$0{\cdot}99976_3$	$0{\cdot}99976_9$	+0·3
„ ...	1160·500	$1{\cdot}00012_8$	$17{\cdot}2438_9$	$0{\cdot}99983_8$	$0{\cdot}99985_5$	+1·7
Jena ...	1138·576	$1{\cdot}00002_9$	$16{\cdot}5965_0$	$0{\cdot}99986_1$	$0{\cdot}99985_8$	−0·3
„ ...	800·722	$1{\cdot}00007_3$	$8{\cdot}2197_8$	$0{\cdot}99967_3$	$0{\cdot}99968_0$	+0·7
„ ...	712·299	$1{\cdot}00000_8$	$6{\cdot}4993_4$	$0{\cdot}99948_0$	$0{\cdot}99946_0$	−2·0
Verre dur	1098·653	$1{\cdot}00007_1$	$15{\cdot}4645_7$	$0{\cdot}99916_5$	$0{\cdot}99917_1$	+0·6
„	1024·146	$1{\cdot}00007_0$	$13{\cdot}4291_3$	$0{\cdot}99987_9$	$0{\cdot}99986_5$	−1·4
„	1049·821	$1{\cdot}00010_0$	$14{\cdot}1116_4$	$0{\cdot}99984_5$	$0{\cdot}99985_6$	+1·1
„	1061·576	$1{\cdot}00013_3$	$14{\cdot}4321_4$	$0{\cdot}99967_6$	$0{\cdot}99967_3$	−0·3

In 1908 an International Conference was held in London, at which the conclusions of the Edinburgh Conference of 1892 were adopted internationally. This Conference for the first time drew a distinction between Absolute and International Units; the Ohm being 10^9 C.G.S. units, the International Ohm the resistance at 0°C. of a column of mercury of uniform cross-section having a length of 106·300 cm. and a mass of 14·4521 gr. In 1910 this became legalised in this country by Order in Council.

At the B.A. meeting of 1889 the name of Viriamu Jones was added to the Electrical Standards Committee. To him are due important modifications in the design and construction of the Lorenz apparatus, which was by that time generally recognised as offering the greatest promise of extreme accuracy in the absolute determination of the ohm. Jones pointed out the importance of construction on an engineering scale, and the great increase in accuracy that would result from the use of single-layer coils of uncovered wire. The double form of Lorenz apparatus at the National Physical Laboratory was built as a memorial to Viriamu Jones. With this apparatus, designed by Mr. F. E. Smith, and used by him, results of extra-ordinary accuracy have been attained. A full account of this work will be found in the "Phil. Trans.," Vol. CCXIV. The conclusion of the experiments is that the Ohm is equal to the resistance of a column of mercury, of uniform cross-section equal to that adopted in the specification of the International Ohm, having a length of 106·245±·004 cm., at the temperature of 0°C.

A full description of the earlier work here outlined will be found in the Collected Reports on Electrical Standards, issued by the B.A. Committee in 1912. Reference may also be made to the Fourth Kelvin Lecture ("Proc." Inst. E.E., 1913.).

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2. "*Two Proposals: (a) Respecting the Present System of Electrical Units; and (b) On a Lecture Demonstration of the Absolute Determination of Resistance,*" *by* F. E. Smith, *O.B.E., F.R.S.*

My remarks are limited to two proposals, one respecting our present system of units, and the other the need for teachers to demonstrate the ease of measuring electrical resistance in C.G.S. measure.

At the present time electrical quantities can be measured in terms of the centimetre, gramme, and second with an accuracy in excess of all commercial requirements. In the case of resistance, as with other electrical quantities, the unit and standard are distinct. The unit of resistance is the ohm, 10^9 centimetres per second, while the standard of resistance, the international ohm, is the resistance of a carefully defined column of mercury at 0°C. In the past the material standard, mercury, has had an advantage over the unit of resistance, the ohm, inasmuch as the accuracy of reproducing the standard was greater than the accuracy with which the unit could be realised. But this is no longer true. The international ohm can be reproduced with an error of about ±2 parts in 100,000, and this has been done in five countries; but the unit of resistance, 10^9 C.G.S. units, can now be realised with an accuracy equally great. Precise measurements of resistance in absolute measure have not been made in many countries or by many observers, but the following results show the accuracy which has been obtained. It should be noted that only the last two observers claim a probable error less than 1 part in 10,000. It has also to be remembered that the errors in reproducing the international ohm are involved in the differences between the values recorded :—

	Observer.	Ohm in centimetres of mercury.
1882	Rayleigh	106·26 cm.
1882	Glazebrook	106·25 ,,
1883	Rayleigh and Sidgwick	106·24 ,,
1885	Wiedemann	106·16 ,,
1889	Dorn	106·24 ,,
1891	V. Jones	106·31 ,,
1912	Campbell	106·27 ,,
1913	Smith	106·245 ,,
1920	Gruneisen and Giebe	106·246 ,,

At the present time it is practically certain that the ohm (10^9 C.G.S. units) can be, and has been, realised with an error not greater than ±2 parts in 100,000. The question arises : "If the international ohm were displaced, and all measurements were made in terms of the ohm, would there be any practical advantage ?" My answer is, Yes. It is clearly desirable that the same physical quantity measured in different ways should have the same value assigned to it. Now, many quantities can be measured by electrical and by mechanical means. As an example, take the frequency of a tuning-fork. If such a frequency is measured through the medium of electrical quantities, and these quantities are expressed in international measure, the result will differ from the value measured by mechanical means by about 5 parts in 10,000. If, however, the electrical quantities were expressed in terms of the absolute system of units, the results would be identical.

My proposal is, therefore, that all resistances should be expressed in ohms (10^9 cm./sec.), without qualification, such as legal, international, &c. Similarly, with the ampere and volt. It is not possible here to deal fully with the advantages and the possible difficulties of change, but the difficulties will certainly increase with time. At an early date I hope to discuss the question much more fully.

My second proposal I will illustrate by means of an experiment.

Teachers of electricity invariably find it difficult to interest students in absolute measurements, and this I believe is largely due to the poor presentation of the subject in text books and the failure to make measurements in front of students, or to encourage students to make measurements for themselves. I submit that it is easy to measure current in terms of the

C.G.S. unit by means of a simply constructed current balance of two coils, or by the use of one coil and a magnet of known moment. But I have never heard of resistance being measured in C.G.S. units as an experiment for students. I hope to show you that the measurement of resistance in terms of length and time is very easy and very instructive; indeed, every student who knows the principle of the dynamo knows the principle of the rotating coil method of measuring resistance. The apparatus consists of a coil of wire of several hundreds of turns and having a diameter of about 30 cm.; it is convenient to divide the coil into two halves, side by side, so that they may be rotated about a vertical axis and yet enable a magnet to be suspended at their centre without disturbance except by electromagnetic action. The original British Association coil is illustrated in many text books, and a simple apparatus based on this can be constructed very cheaply. The arithmetic of the rotating coil method is very simple. Here it is:—

a=Radius of coil.

A=Area of one turn$=\pi a^2$.

n=Number of turns of wire.

N=Number of revolutions per second.

$\omega=2\pi N$.

θ=Angle the plane of the coil makes with the magnetic meridian. $\theta=\omega t$.

H=Horizontal intensity of earth's magnetic field.

R=Resistance of coil.

Self-inductance is neglected.

At time t the flux through coil$=nAH\sin\omega t$.

Induced E.M.F.=rate of change of flux$=-nAH\omega\cos\omega t$.

i the current at time $t=\dfrac{-nAH\omega\cos\omega t}{R}$.

At time t the magnetic force at right angles to the meridian$=\dfrac{-2\pi ni}{a}\cos\omega t$.

Substituting for i, force$=\dfrac{-2\pi n^2AH\omega}{2aR}(1+\cos 2\omega t)$.

The first part of this is constant; the other portion is periodic, and may be neglected when the frequency is great.

If φ=angle of deflection of magnet,

$$\tan\varphi=\frac{2\pi n^2AH\omega}{2aRH}=\frac{2\pi^3n^2aN}{R}.$$

$$\therefore\quad R=\frac{2\pi^3n^2aN}{\tan\varphi}.$$

In practice the coil is rotated in the magnetic field of the earth; a current is induced in the coils and the magnet is deflected by the action of the current. The final formula for the resistance of the coil involves the radius of the coil, which is a length, and the number of revolutions per second, which has the dimensions of T^{-1}; *i.e.*, the dimensions of resistance are the same as those of a velocity.

I submit that the theory of the rotating coil method is very simple and the essential parts of the apparatus are easy to make. A measurement for demonstration purposes takes from two to three minutes, and I propose, therefore, that teachers should seriously consider introducing such an experiment to illustrate their lectures.

[Mr. Smith made an absolute measurement of resistance in front of the audience with the original British Association apparatus.]

II.

INSTRUMENTS BASED ON THE TEMPERATURE VARIATION OF RESISTANCE.

Resistance Thermometry, &c.

1. "*Platinum Thermometers, a Compensated Bridge, a Compensated Bolometer, and the Radio-Balance,*" *by Prof.* H. L. Callendar, *LL.D., F.R.S.*

Prof. H. L. Callendar exhibited and explained some platinum thermometers, a compensated bridge, and some instruments for measuring radiation.

(1) *Calorimetric Thermometers.*—A very sensitive platinum thermometer was exhibited, made for Drs. Lea and Gaskell in 1887, for measuring the heat developed by the beating of a frog's heart. This was one of a differential pair, each consisting of 8 ohms of pure platinum wire, a thousandth of an inch in diameter, wound on a very thin plate of mica 3 mm. square, and insulated with thin plates of mica attached on either side with a special cement. This type of thermometer was described and figured in the British Patent No. 14,509, 1887, and proved to be extremely quick and sensitive for delicate calorimetric work at ordinary temperatures. Similar thermometers, of various sizes, with flat coils wound on mica, enclosed in glass, metal or porcelain sheaths, were made for other experiments about the same time, but none of these have survived.

(2) *Compound Thermometers for Linear Scale.*—The experiments with the gas-thermometer (Phil. Trans. A., 1887) showed that the variation of resistance of pure metals with temperature was of a different type to that deduced by Siemens from his experiments with the copper-ball pyrometer. The coefficient of t^2, or the difference-coefficient, being positive for some metals and negative for others, it was possible, as explained in the patent, by combining two metals, to make a compound thermometer giving a linear scale of temperature over an extended range. This device has since proved useful in special cases, but for accurate thermometry it is better to use a single pure metal, such as platinum, for the thermometer, and to reduce the readings to the gas-scale by the difference-formula.

(3) *Compensated Resistance Box.*—The same patent described the compensation of the coils of the resistance box for changes of atmospheric temperature, by means of a series of compensating coils in the opposite arm of the bridge. The box exhibited at the meeting was one made in 1893, and used in the experiments on the variation of the specific heat of water, and in other investigations requiring a high order of precision. The resistance coils were made of platinum-silver, and the compensating coils of platinum, both carefully annealed. Each pair of coils was separately adjusted to a constant difference over the range 10 to 30°C. before being placed in the box. This method of eliminating the correction for variation of box-temperature saves a great deal of trouble in work of precision, and is preferable to assuming a common temperature, or a common temperature-coefficient, for the various coils.

(4) *Compensation of Thermometers for Conduction along the Leads.*—A compensated platinum thermometer was exhibited, which was used at McGill College in 1895 for observing rapid variations of temperature in the cylinder of a steam-engine, in determining the adiabatic index of dry steam, and investigating the phenomena of supersaturation. About a dozen thermometers of this type were made for these experiments.* Each thermometer consisted of two loops of pure platinum wire, 0·001 in. in diameter, fused autogenously to two pairs of thick leads of platinum projecting from a glass-stem, in the end of which they were hermetically sealed. The fine wire was directly exposed to the steam and followed all the variations of temperature with almost incredible quickness. The two loops were connected on opposite sides of the bridge-wire, so that the observation of the balance-point gave the difference of resistance corresponding to the central portion of the longer loop, automatically corrected for end-effects due to conduction along the thick leads. Without this provision it would have been impossible to obtain satisfactory observations with the temperature varying 200 deg. or 300 deg. in each cycle at the rate of 100 revs. per min. The temperature at any point of the cycle was observed by setting a periodic contact to close the circuit of the galvanometer at the desired point. The advantage

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* The fundamental interval of each of these thermometers was adjusted within 2 or 3 parts in 1,000 to 2 ohms, a fine adjustment being obtained by slightly stretching either one loop or the other as required.

of the method for measuring the adiabatic index of steam was that errors due to leakage or condensation on the walls did not affect the relation between pressure and temperature in the middle of the cylinder, whereas they produced marked distortion in the pressure-volume relation shown on the indicator card.

The same method of compensation for end-effects was employed for many similar purposes, *e.g.*, in investigating the heat-losses from heated platinum wires, the melting-point of platinum and other substances with a resistance meldometer, and the variation of the viscosity of gases by the transpiration method at high temperatures. (Proc. R.I., 1899.)

(5) *Compensated Bolometer.*—An important application of the same principle was made in the construction of a bolometer for the absolute measurement of radiation by Kurlbaum's method. In this method the rise of temperature of the grid when exposed to radiation is first observed. The radiation is then cut off, and the increase of current required to produce the same rise of temperature is measured. As applied by Kurlbaum, this method gave a result for the constant of radiation about 7 or 8 per cent. too low. It appeared probable that the error was to be attributed to a difference in the end-effects in the two cases. A compensated bolometer was accordingly constructed with two similar grids of different lengths, used differentially, as described and exhibited at a meeting of the Physical Society in May, 1905. The area exposed to radiation was defined by a rectangular aperture corresponding to the difference in length of the grids. This gave a result free from end-effects, agreeing closely with recent measurements of the constant, and was confirmed by the entirely independent method of the radiobalance.

(6) *The Radiobalance.*—The defects of the bolometric method for the absolute measurement of radiation are, (1) that the electric compensation is not simultaneous, but requires successive observations; (2) that the absorption coefficient of the grid is somewhat uncertain and difficult to determine. In the radiobalance, these objections are met, (1) by simultaneous compensation of the radiation by absorption due to the Peltier effect in a thermo-junction through which a measured current is passed; (2) by using a cup to receive the radiation admitted through a small circular aperture in such a way that the absorption is practically

complete. Four types of radiobalance were exhibited: (1) a laboratory instrument, with water or steam-jackets, for investigating the variation of the coefficients over large ranges of temperature; (2) an instrument with an equatorial mounting for convenience in observing solar radiation; (3) a similar instrument with an altazimuth mounting, which is more convenient for laboratory work, but can be used on the sun without difficulty; (4) a radiobalance in a simple block mounting, which could be used either vertically or horizontally, and was specially adapted for delicate calorimetric experiments, such as the direct measurement of the Peltier effect, or of the heat-emission of small quantities of radium. This instrument was employed at the meeting to demonstrate the heat-emission from a bulb containing about 1·8 mgm. of radium. There is no difficulty in measuring the heat-emission, by balancing it with the heat-absorption due to the Peltier effect, to a tenth of a microwatt, or better than one in a thousand, for a single milligram of radium.

When tested on radiation measurement, all the radiobalances gave results consistent with each other to a similar order of accuracy, which is about the limit to which a given intensity of radiation can be reproduced, either with a focus-lamp, or from a black body electrically heated. The statement made by W. W. Coblentz (Bull. U.S.A. Bureau, 1913) that the method was of no value for absolute measurement, and that two of the results given by the author (Proc. Phys. Soc., XXIII., Dec., 1910) differed by 12 per cent., was very misleading, because the two results quoted referred to separate tests with different focus-lamps at different distances, and were in no way comparable. Unfortunately, the Bureau omitted to send the author a copy of the Bulletin containing this criticism, so that he was unable to correct the misstatement at the time.

2. "*Early Work on the Resistance Pyrometer*," *by* C. R. Darling, *F.I.C.*

I DESIRE to refer briefly to the early work of Sir William Siemens, who was the first to measure temperatures by the resistance method. Siemens constructed a simple resistance thermometer of copper wire in 1860, which was used for measuring the rise of temperature in the interior of cables, and described the instrument before the British Association at Manchester in 1861. Subsequently, with a view to extending the method to the measurement of furnace temperatures, Siemens investigated the change in the resistance of platinum with temperature, using mercury thermometers as standards up to 350°C., and a calorimetric pyrometer for higher temperatures. He concluded that the relation between temperature and resistance could be expressed by a parabolic formula, in which zero resistance corresponded with the absolute zero of temperature. A practical form of pyrometer, based on the data thus obtained, was made in 1871, and used to a small extent industrially. The coil of platinum was wound on a pipeclay former, and a third lead was used to compensate for variations in the resistance of the ordinary leads due to varying depth of immersion. Experience in the use of this pyrometer showed that the coil should be adequately protected from furnace gases, as otherwise a serious alteration in the zero resistance occurred. Resistance pyrometers for commercial use have been made by the firm of Siemens & Co. since 1871, three forms of indicating apparatus having been successively used, viz.: (1) A differential voltameter, in which the resistance of the pyrometer coil was deduced from the volumes of gas liberated in two tubes containing dilute H_2SO_4, and each forming part of a differential circuit; (2) an ordinary differential galvanometer circuit, in which the pyrometer coil was balanced against an adjustable resistance; and (3) a Wheatstone bridge arrangement, with direct temperature readings. Of these, the last-named is the only one in use at the present time. The pipeclay former is still employed, and the coil is surrounded by magnesia to prevent access of furnace gases.

(Examples of the three types of indicator mentioned above, and a coil wound on pipeclay, were exhibited and explained by Mr. Darling.)

3. "*Resistance Thermometers for Steam Temperatures*," *by* C. Jakeman (*National Physical Laboratory*).

In connection with experiments upon the properties of steam under pressure it was found convenient to use platinum resistance thermometers with envelopes of steel tube arranged so that the metal envelope could be screwed into a special fitting in the steam pipe. These thermometers were of 1 ohm. "fundamental interval" and the steel envelopes were about 10 mm. in diameter. For a large range of work these thermometers proved to be somewhat deficient because there was considerable time lag in taking up the temperature owing to the mass of the metal in the envelope and its high conductivity, and the size of the thermometers prevented their use in small pieces of apparatus.

An attempt was made to construct a thermometer of similar form and range but with much reduced heat capacity and overall diameter. For the work in hand a temperature of 400°C. was the highest required, and that only occasionally, but at the same time it was necessary to make the thermometer so that its sulphur-point could be determined for calibration. The envelope was constructed of thin brass tube 3 mm. external diameter closed at the lower end with a brass plug silver-soldered in place. The internal diameter of the tube was a little less than 2·5 mm., and the provision of a platinum coil of usual length and diameter would give a bulb of inordinate length. The diameter of the platinum wire used was 0·025mm. and a simple loop containing 1·7 cm. of wire is all that is necessary to obtain a fundamental interval of 1 ohm. The insulation of the leads in the tube was provided by four glass capillary tubes each fitting over one of the four leads. The leads were of copper 0·25 mm. in diameter and the joint between the thermometer leads and the loop of platinum wire was made by autogenously welding the two metals together in a very small oxy-coal gas flame. This requires some care, but with practice a perfect joint can be made with a very small ball of metal at the joint. These joints were then drawn back into the capillary tubes and the glass closed round the metal ball, thus preventing the wires sliding in the glass tubes. (The joint between the copper lead and platinum loop can be made by silver soldering; but it is difficult to make the joint small enough to enter the capillary in this case.) The head was of ordinary construction, the resistance of the pair of copper compensating leads being made equal to that of the pair of thermometer leads and the compensating and thermometer flexible leads being similarly balanced. The insulation thus provided is satisfactory, but there is still considerable space in the tube which allows convection currents to form in the air enclosed. These were prevented by filling the lower part of the envelope with flakes of mica.

The thermometer proved to take up the surrounding temperature extremely rapidly, being better than a mercury in glass thermometer, while the diameter of the envelope enables it to be used in any place in which there is room for a small size mercury in glass thermometer.

Some of these thermometers have been in use in the engineering laboratory of the National Physical Laboratory for some 10 years and have proved very satisfactory. Owing to the method of construction of the loop it has not been found possible to adjust the resistance with any accuracy and the welding of the joints is a somewhat uncertain process, so that the fundamental interval may be ±20 per cent. of that required; and if direct readings are required a specially calibrated bridge must be used. Owing to the small size of the wire the current density must be small and it is usual to use 0·002 amperes. If the current is more than this heating of the loop is apparent in a very few seconds. With a Broca galvanometer of moderate sensitivity, readings can be taken to 0·01°C., which is sufficient for most experimental work.

One of these thermometers was exhibited, and also a model in glass showing the insulation of the leads and the loop resistance

4. "*An Electric Transmitting Radiator Thermometer,*" *by*

EDGAR A. GRIFFITHS.

THE instrument described below may be of interest as it is commonly supposed that resistance thermometers find no application on the modern aeroplane.

This outfit was captured during the war and formed part of the equipment of a bombing machine.

As no published description of the instrument could be found it was taken apart and carefully investigated.

The outfit is used for measuring the temperature of the water in the engine radiators, and two pyrometers were connected to one indicator fixed on the instrument board.

The indicator operates on the same principle as the ohmmeter.

The resistance thermometer is made of platinum wire, wound on a strip of micanite. The coil is enclosed in a flat copper tube, forming part of a brass elbow which screws into the radiator. The resistance of the thermometer coil at zero centigrade is about 90 ohms; this high resistance is desirable in view of the fact that no compensation is applied for the change in resistance of the connecting leads and the variation in these leads would be important with a low resistance thermometer. A push-button switch forms part of the equipment, so that the current is only used during the actual observation.

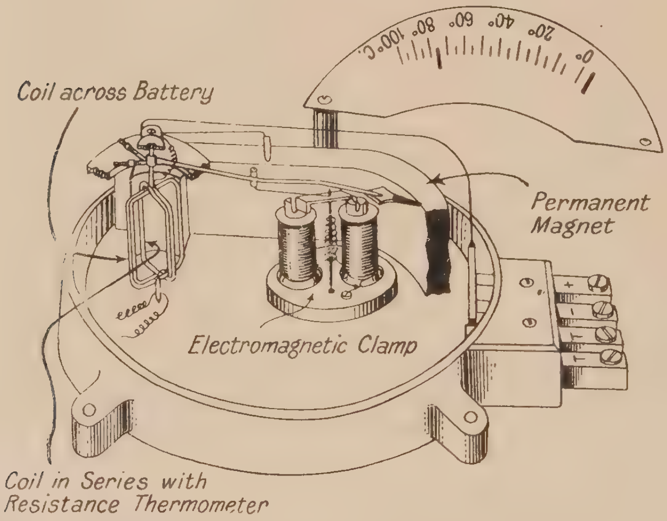

Fig. 1.—View of Instrument.

The indicator (Fig. 1) is a miniature form of the well-known simple ohmmeter; it is based on the moving coil principle. The two coils are wound crosswise on the same aluminium frame, and move freely in a powerful magnetic field. One coil is connected through a resistance across the battery (*see* Fig. 2), and circuit connections may be traced from the battery by conductors 1 and 10, resistance 11, conductor 12, coil 13, conductor 7, resistance 8, and conductor 9 to the battery. The other coil is connected in series with the resistance thermometer. Circuit connections may be traced from the battery by conductors 1 and 2, resistance 3, conductor 4, through the thermometer, by conductor 5, coil 6, conductor 7, resistance 8, and conductor 9 to the battery.

The indication is thus the ratio $\frac{\text{electromotive force}}{\text{current}}$, *i.e.*, the resistance of the thermometer circuit. The reading is theoretically independent of the pressure employed, but actually the E.M.F. must be kept within limits: otherwise, if the E.M.F. is too low, friction decreases the sensitivity, and if too high, the current causes appreciable heating up of the resistance thermometer above its surroundings.

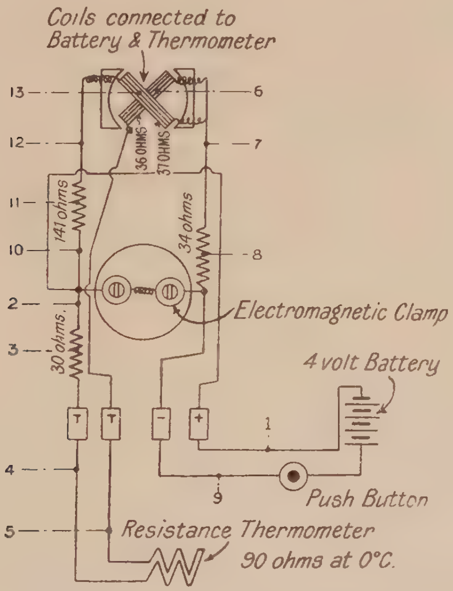

Fig. 2.—Diagram of Connections.

The same rule should, in fact, be used here as in the case of the laboratory platinum thermometer; the current should always have the same value as that used in the calibration of the instrument.

It will be evident from the principle of this instrument that, owing to the absence of any control spring, the pointer always remains at a position somewhere away from the zero, depending on the last reading taken: this apparent reading would prove somewhat deceptive and therefore requires to be eliminated by some form of clamp for setting the pointer back to zero. The clamp in this particular instrument consists of a small arm, pivoted about a vertical axis, and arranged to hold the pointer at zero by the action of a spring. When the current is switched on, the arm is immediately swung round to the top of the scale by a small electro-magnet connected across the battery, and remains fixed while the reading is taken. As soon as the current is switched off again the arm is released and returns the pointer to zero.

This device will be clear from Fig. 2. The scale of the instrument is very cramped, owing chiefly to the large angle subtended by the crossed coils (60 degrees in this case). The graduation is, moreover, not uniform, but closed up in the central part of the scale, making the calibration somewhat troublesome. It would, of course, be an improvement to use a smaller range of temperature for the purpose of controlling the temperature of the water in the radiator (instead of 0° C. to 100° C., as in this instrument).

The Hot-Wire Microphone.

"*Experiments Illustrating the Function of Convection Currents in Determining the Sensitivity of the Hot-Wire Microphone,*" *by Major* W. S. TUCKER, *R.E., D.Sc.*

THE hot-wire microphone is essentially an instrument for recording alternating air currents by its change in electrical resistance, and a full description of it has been given in a paper on "The Selective Hot-Wire Microphone" read before the Royal Society on January 20 of this year.

It has been found by experiment that when a hot-wire grid is inserted in a diaphragm so that it intercepts progressive sound waves, it is too insensitive to give satisfactory electrical indications unless the sound is very intense. For this purpose, however, the air must be very still, as the smallest pressure difference on the two sides of the diaphragm causes air currents which completely spoil the effect.

If, however, the rear side of the diaphragm be a closed cavity, and if, furthermore, the diaphragm aperture and the cavity together be made to constitute a Helmholtz resonator, the new arrangement serves not only to protect the wire from draughts, but if correctly tuned, will give considerable magnification of the sound received. The tuning of this resonator is effected by alteration of orifice or volume of the cavity as expressed by the well-known relation—

$$n = \frac{a}{2\pi}\sqrt{\frac{c}{S}}$$

where a is the velocity of sound, S the volume of the cavity and c a quantity called by Rayleigh the "conductivity of the orifice."

The resonance curve for the completed microphone, consisting of Helmholtz resonator and hot platinum grid, is fairly sharp, depending on the nature of the aperture and the material of the resonator.

The resonators of the microphones hitherto employed give a magnification of from 50 to 100.

The effect of convection currents on the sensitivity of the hot wire for responding to oscillatory air currents can best be illustrated by a study of what may be termed the "characteristic curve" of a hot wire.

This curve gives the relation between resistance and convection current for a hot wire in which the electric current is kept constant.

Such a curve was obtained by Capt. Paris, employing air currents of velocity less than 6 cm. per second and with a wire of diameter 0·0006 cm.

For observations of alternating air currents the above quantities are most suitable, but as they employ different conditions to those employed by L. V. King* or J. S. G. Thomas,† it was thought desirable to take new measurements.

The current of air in the orifice was controlled by an aspirator with suitable stop-cock, and the velocity of the imposed current, which is the average over the microphone aperture, could be calculated. The curve shown, which is already contained in the Paper above mentioned, shows a typical "characteristic curve." The positive values of V, the air velocity, oppose in direction the convection due to the wire.

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* L. V. King, "Phil. Trans.," A. Vol. CCXIV., pp. 373-432.

† J. S. G. Thomas, "Phil. Mag.," Vol. XXXIX., pp. 505-534.

At Q the temperature of the wire indicated by its resistance is the same as at O, so that the value OQ is twice the velocity of the convection current. At N when $ON=\frac{1}{2}OQ$ the temperature acquires its maximum at a point where the air in the orifice is at rest. As V extends beyond the limits of O and Q the curve is extended and proves to be very nearly parabolic for small values of V.

The sensitivity of the hot wire for a given sound is measured by $\frac{\partial(\delta R)}{\partial V}$, and it is obvious that this increases approximately with the velocity of flow. This can be shown by experiment. It involves opening the walls of the Helmholtz cavity to admit a current of air. Such an aperture alters the pitch of the resonator and the volume must be adjusted to correct for it.

Since the resonance curve for the resonator is so sharp, it is essential that a preliminary test should be made to find out if the pitch of the resonator is altered by passage of the air current. No such alteration was observed.

If now an air current be passed through the resonator while it is responding to a steady source of sound, variation of sensitivity is shown in agreement with the characteristic curve.

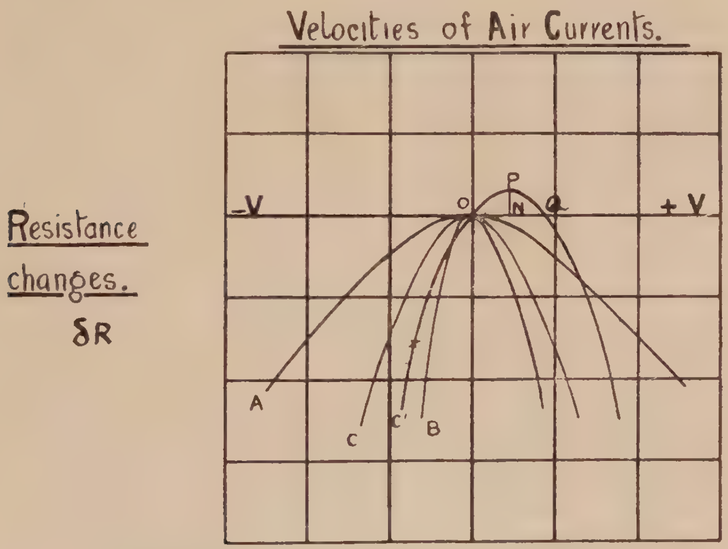
There is, however, a limit to the magnitude of the superimposed air current, so that magnification cannot be produced indefinitely. This may be demonstrated by passing the microphone current through an amplifier, such as that employed by the Army. If the effect be observed by a telephone the smooth sound of the tuning fork source, which may be employed, becomes broken up and gives a crackling sound. This effect is probably due to the production of swirls round the hot wire, and it is noticeable that these swirls are produced by air currents which are comparatively small.

The amount of magnification obtainable may be measured by a vibration galvanometer tuned up to the frequency of the tuning fork, and it is then seen that the deflection actually diminishes again after the superimposed current has attained a critical value. This may be due to the fact that the swirls in the region of the wire protect it from the vibratory motion of the air in the orifice.

The manner in which the hot-wire interprets the alternating air current movements depends on its condition as represented by a point on the characteristic curve. Thus, at

R an effect of the same periodicity is observed. At *P* the hot wire acts as a "rectifier," giving only a cooling effect whether the velocities of the vibrating air particles be positive or negative. It should be noted that the fluctuation in microphone current has twice the periodicity of the sound which produces it, so that, if the conditions of the hot-wire be indicated by *P* the octave will be reproduced.

If the condition of the hot-wire be represented by a point near *P* the sound reproduced will contain the fundamental and the octave together when the intensity of the sound is sufficiently large.



A—Coarse wire. B—Fine wire. CC′—Medium Wire.

Curve *C* in the diagram would be obtained for a microphone wire in the horizontal plane, and the alternating air currents which it measures have displacements in the same direction as the convection currents.

If, however, the resonator be gradually rotated until the hot wire is brought into a vertical plane, it is only the resolved part of this convection current which is effective in increasing the sensitivity and the curve gradually passes from condition C_1 to *C*. With no superimposed convection current, therefore, the octave of the original note is reproduced and not the fundamental.

"Characteristics" for wires of different diameters are difficult to compare. It should be laid down that the maximum temperatures and the total resistance at these maximum temperatures should be the same and that they should be similarly mounted in the same orifice.

Characteristic curves will always be symmetrical about the axis of resistance, and it can be shown that the curvature for coarse wires will be less than for fine ones. (Curves A and B.)

It is expected, therefore, that variation of the superimposed convection current will produce less effect on the coarser than the finer wires.

No attempt is made to compare by means of these curves the sensitivities of the different wires for a given sound under the same convection conditions, since account has to be taken of the thermal capacities and thermal lag of the wires. This is a much more difficult problem, which is now receiving consideration.

Hot Wire Anemometers.

1. "*A Hot Wire Anemometer*," *by Prof.* J. T. McGregor Morris, *M.I.E.E.*

Prof. Morris demonstrated the action of the instrument devised by him, an account of which may be found in the following Papers:—

(*a*) "The Electrical Measurement of Wind Velocity." Dundee B.A. "Engineering," Dec. 27, 1912.

(*b*) "The Distribution of Wind Velocity in the Space Surrounding a Circular Rod in a Uniform Current of Air." "Engineering," Aug. 8, 1913.

(*c*) "A Portable Direct-Reading Anemometer for the Measurement of Ventilation in Coal Mines." Cardiff B.A. "Electrician," Aug. 27, 1920.

2. "*The Directional Hot-Wire Anemometer*," *by* J. S. G. THOMAS, *D.Sc.* (*Senior Physicist, South Metropolitan Gas Company, London*).

THE laws of convection of heat from fine heated wires are such as to indicate the hot-wire type of anemometer as more especially the type of instrument for the precise measurement of the velocity of a slow moving stream of fluid. In the region of low velocities, difficulties arise in the application of the types of hot-wire instruments devised by King* and Morris† respectively, owing to the disturbing influence upon the convection effect due to the stream, produced by the free convection current arising from the heated wire exposed to the cooling action of the stream. The effect of such free convection current in the region of low impressed velocities of the stream is readily seen from Fig. 1, in which are given the respective forms of the calibration curves obtained by employing a hot-wire anemometer of the Morris type in the investigation of the flow of air in a brass pipe of diameter 2·04 cms. The hot wire was approximately 0·1 mm. diameter, and the heating current 1 000 ampere. The calibration curves were determined for both upward and downward flows of the impressed air stream, and in each case calibrations were made with the shielded and unshielded wires respectively leading. It will be seen from the figure that the calibration curves for velocities greater than about 5 cms. per second were slightly different according as the shielded or unshielded wire was the first to meet the flow. This result was probably due to turbulent conditions of flow produced when the stream was first incident upon the unshielded wire. It is not proposed to discuss this matter further here, but it may be remarked that the results indicate the power of the hot-wire instrument in the investigation of turbulent conditions of flow—a matter to which attention has been drawn by King.‡

It will be seen from Fig. 1 that whereas the respective calibration curves for upward and downward flow of the impressed air stream, with the same wire leading in each case, are identical for impressed velocities greater than about 20 cms. per second, a marked difference in the calibration curves exists for velocities below this critical value in the present instance. This is most clearly seen by a consideration of the portions of the respective calibration curves corresponding to impressed velocities between 0 and 6 cms. per second. Whereas in the case of an upward flow, the deflection is positive and increases with increase of the impressed velocity between these limits, the deflection in the case of the downward flow is initially negative and increases numerically to a maximum value, thereafter diminishing to its initial zero value. Employing the instrument for the investigation of the velocities of stream at various inclinations to the horizontal

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* "Phil. Trans." Roy. Soc., A.520, 214, pp. 373-432 (1914). Eng. Pat. 18,563/1914.

† British Association, Dundee, 1912. "Electrician," Oct. 4, 1912, 1056. Eng. Pat. 25,923/1913.

‡ "Phil. Trans.," *loc. cit.* p. 407.

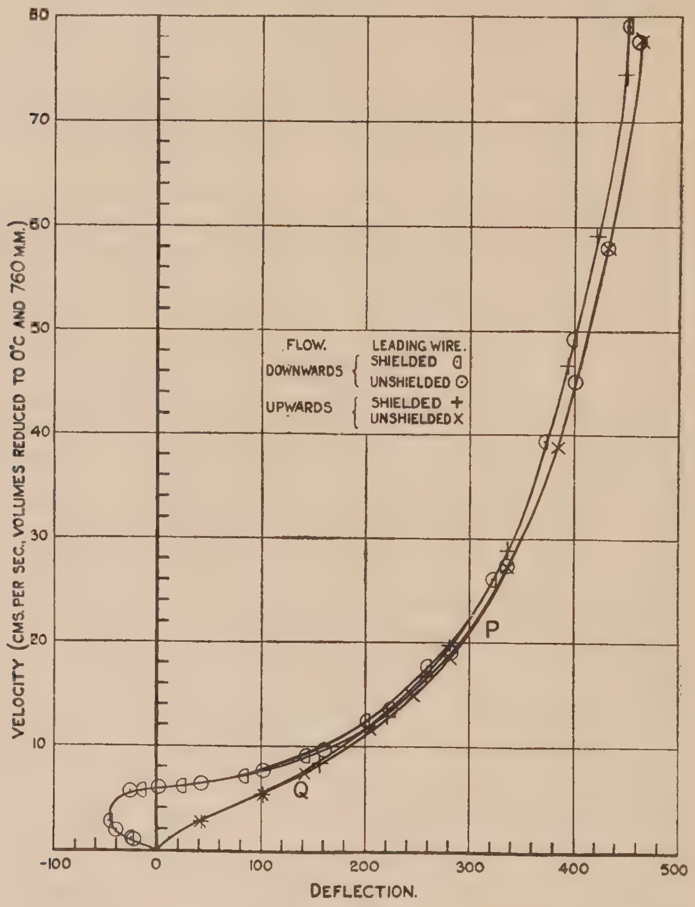

Fig. 1.

using the heated wire in a horizontal position throughout, various forms of the calibration curve may be obtained in the region of low velocities, such forms being enclosed within the two corresponding to upward and downward flow shown in Fig. 1. Various other peculiarities of the calibration curves for the hot-wire type of instrument referred to have been investigated by the author, and a type of directional instrument devised in which the difficulties originating in the free convection current have been almost entirely eliminated. In the directional type of instrument, two fine platinum wires are mounted parallel, and one behind the other in close juxtaposition, transversely to the direction of flow of the gas in the pipe or channel. The wires are inserted in a Wheatstone bridge in the usual manner, and the indications of the instrument enable both the magnitude and direction of a slow-moving stream of fluid to be readily ascertained. The characteristics of such a type of hot-wire instrument, and the influence of the free convection-current upon the indications of the hot-wire amemometer have been discussed by the author in the following series of Papers :—

(1) "Hot Wire Anemometry: Its Principles and Applications." Journ. Soc. Chem. Ind., 1918, XXXVII., 165T-170T.
(2) "The Hot Wire Anemometer: Its Application to the Investigation of the Velocity of Gases in Pipes." "Phil. Mag.," 1920, XXXIX., 505-534.
(3) "A Directional Hot Wire Anemometer of High Sensitivity, especially applicable to the Investigation of Low Rates of Flow of Gases." Proc. Phys. Soc., 1920, XXXII. (III.), 196-207.
(4) "The Directional Hot Wire Anemometer: Its Sensitivity and Range of Application." Phil. Mag., 1920, XL., 640-665.
(5) "The Thermal Effect produced by a Slow Current of Air flowing past a Series of Fine Heated Platinum Wires, and Its Application to the Construction of Hot Wire Anemometers of High Sensitivity, especially applicable to the Investigation of Slow Rates of Flow of Gases." Phil. Mag., 1921, XLI., 240-258.
(6) "A Null-Deflection Constant-Current Type of Hot Wire Anemometer for Use in the Determination of Slow Rates of Flow of Gases, together with an Investigation of the Effect of the Free Convection Current upon such Determinations." Phil. Mag., 1921.

A type of hot wire anemometer utilising the effect referred to in the fifth of the series of Papers given above has been described by H. Gerdien and R. Holm,[*] who likewise describe an obstructive device ("stauscheibe") for insertion in the flow tube, whereby the instrument may be employed for the measurement of large velocities of gas streams.

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[*] "Wiss. Veröffentlichungen aus dem Siemens-Konzern," Bd. I., Heft I., pp. 107-121 (1920).

3. "*An Instrument for Use in Measuring Convected Heat,*" *by* ALFRED H. DAVIS, *B.Sc.* (*National Physical Laboratory*).

INTRODUCTION.

HEAT losses by convection depend not only on the area of the surface of the body, but also on its form and orientation. If the theoretical calculation were possible in a few ideal cases, in actual practice it would still often be necessary to determine the heat loss by other means. Even in ideal cases the laws of convection are not yet accurately known, and it is desirable to ascertain the circumstances of air temperature and velocity in the vicinity of the cooling body. In this connection the present apparatus was devised to trace convection losses in the air itself.

If air, of density ρ and temperature θ, flowing with velocity v, was originally at 0°C., then the air has been supplied with heat which is passing a section normal to the stream at the rate of

$$v\rho s\theta \text{ cals. per sq. cm. per sec.} \quad \ldots \quad (1)$$

s being the known specific heat at constant pressure. The instrument to be described is a hot-wire anemometer measuring $v\rho$,* independent of the temperature of the air, and adapted for use as a resistance thermometer to obtain θ. Thus it determines the rate at which convected heat is passing a given point.

It was constructed in two types, one being a null instrument, the other being direct reading, and so more convenient, but involving an approximation which somewhat restricts its range.

PRELIMINARY THEORY OF HOT-WIRE ANEMOMETER.

King,† in comprehensive experiments with platinum wires, verified his theoretical formula of the form

$$H/\theta = B\sqrt{(v\rho)} + C \quad \ldots \quad (2)$$

where H=heat loss per unit length of wire.

θ=*Excess* of wire temperature over air stream temperature.

v=velocity of air.

ρ=density of air.

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* In calibrating the instruments, mass flow $v\rho$ was expressed in terms of the velocity v_{17} at which air at 17°C. would have to move to give the same mass flow.

Thus, $v\rho = v_{17}\rho_{17}$; and $v\rho s\theta = 0{\cdot}000289\ (v_{17}\theta)$ cals./cm.^{2}/sec. . . (1A)

† King, "Phil. Trans.," 214, p. 373, 1914.

B and C being constant for a given wire and temperature excess, and having but a small coefficient of change with temperature excess.

In his experiments, although the temperature of the wire was altered, that of the ambient air was always about 17°C.; but theoretically there is no reason for appreciable departure from the law through change of actual air temperature, for temperature *excess* is the essential factor. The null instrument was arranged to work at a constant temperature excess, and by experiment in a wind channel to which heated air was supplied, it was found that in these circumstances the relation between H and $v\rho$ was independent of the actual temperature of the air.

DESCRIPTION OF THE NULL INSTRUMENT.

Construction.

The instrument consists essentially of a Wheatstone's net, as is seen from Fig. 1. In the fourth arm of the net TT is the 6 mil platinum wire, which is placed in the air stream. An ammeter, A, gives the current through the wire. Manganin is the material of the rest of the bridge, as this has no temperature coefficient. SS is a slide wire. The third arm of the net consists of two coils, one of which can be short-circuited as desired by the key K, which carries an indicating pointer. For convenience, this pointer also controls the sensitivity of the galvanometer, and further makes about a hundredfold

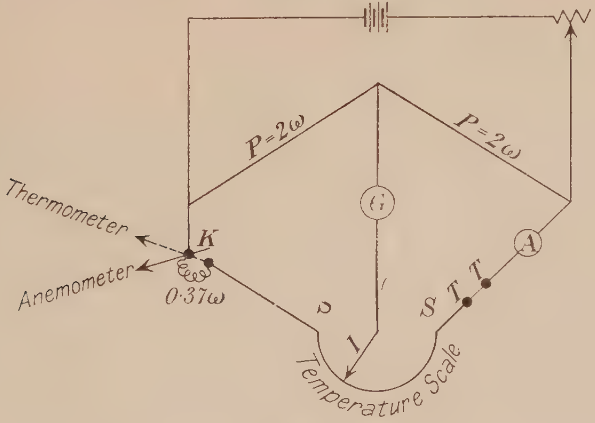

FIG. 1.

difference to the current in the bridge. The connections, however, are not shown.

Method of Use and Theory of Null Instrument.

Setting the indicator at *Thermometer* closes the key K and passes but a small current through the bridge. The instrument is now a resistance thermometer, the temperature of the air being read directly on the bridge wire scale from the position of the pointer I for balance. The slider is fixed in this position.

Setting the indicator at *Anemometer* now passes a larger current through the bridge and opens the key K, thus increasing the resistance of the arm by 0·37 ohms. Balance can only be restored by heating the platinum wire to a definite temperature excess (108°C.), independent of the initial temperature of the air.* The requisite heating current i, which restores balance, is read from the ammeter. If R is the resistance of the wire at its working temperature, we have, using (2),

$$i^2R/\theta=H/\theta=B\sqrt{(v\rho)}+C \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

or, since θ is constant,†

$$i^2R=F(v\rho). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Thus i^2R measures the mass flow of air, and noting that the position of the pointer I already determined along the slide wire gives R, we see that i^2R is readily obtained from one setting of a slide rule, and the mass flow is then known.

Even more conveniently, a three-scale nomogram may be readily constructed in which a thread stretched between the value θ on one scale and i on another cuts the third scale at a point from which the air velocity is read off directly.

Calibration of the Instrument.

As thermometer the instrument was calibrated by direct determination of the ice point and of the steam point, and checked from a knowledge of the constants of the bridge wire and of the platinum wire.

As anemometer it was calibrated in wind channels. For use in measuring convection currents the calibration should extend as far as possible into the region of low speed. Also, when the air speeds become so low that they are comparable with the speed of the natural convection current from the anemometer wire, the instrument should be calibrated under such conditions that the air velocity is in the same direction relative to the vertical, as in the experiment in which the instrument is to be employed.

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* This is exactly true when the resistance-temperature law is linear.

† Rigorous constancy of θ is not essential, for if θ depends on the initial temperature of the wire, the position of the pointer I determines R/θ, and (3) is used instead of (4) without complication.

The actual instrument worked at 108°C. temperature excess, but some preliminary work was done at air temperatures with an anemometer working at nearly three times this excess, so that the natural convection currents should be more pronounced. In this, instead of using vertically upward currents, the instrument was moved downwards through still air by a modification of Attwood's machine. The calibration was found to agree with that for horizontal currents in a wind channel, except, perhaps, for speeds lower than 10 cm. per second, and the relation (2) held down to 15 cm. per second, needing slight correction below.

The actual instrument should be reliable at least to these limits, and it was calibrated for various temperatures of air stream in a specially constructed vertical channel, and for room temperatures it was also calibrated in the horizontal 3 ft. wind channel of the National Physical Laboratory. In the horizontal channel the usual laboratory calibration was used for the higher speeds; however, since the pressure head measurements become insufficiently accurate for speeds less than 100 cm. per second, the lower speeds were obtained by observing smoke puffs through a window, and timing them with a stop watch over a distance of about 1 metre.

The special vertical channel is shown diagrammatically in Fig. 2. The entrance contained layers of wire gauze, and a honeycomb grid to break up eddies and to ensure an evenly distributed and vertical flow of air. The speeds being too slow for direct measurement with the Pitot tube, this was inserted in a subsequent portion of the channel of smaller diameter, reached by a section of approximate streamline shape. For higher temperatures over a range of about 50°C., the air entering this channel was heated by the heater grid. The distribution of velocity across the neck was determined by the Pitot tube, and found to be independent of the temperature of the air stream. It was found that the calibration for ordinary temperatures in the horizontal channel agreed with those for the vertical channel and the hotter air, up to the temperatures reached. The results are shown graphically in Fig. 3.

The relation (2) holds down to 9 cm. per second, and is not found to depend on the temperature of the ambient air.

THE DIRECT READING INSTRUMENT.

Direct-reading instruments have already been used,* in which, for ordinary temperatures, the out-of-balance deflection measures the mass flow of air. In the instrument here described it has been found possible to allow for the effect of temperature very approximately by choosing for each temperature an appropriate current through anemometer wire.† The instrument differs from Fig. 1 only in the absence of the 0·37 ohm coil and its key K. The indicator which carried K

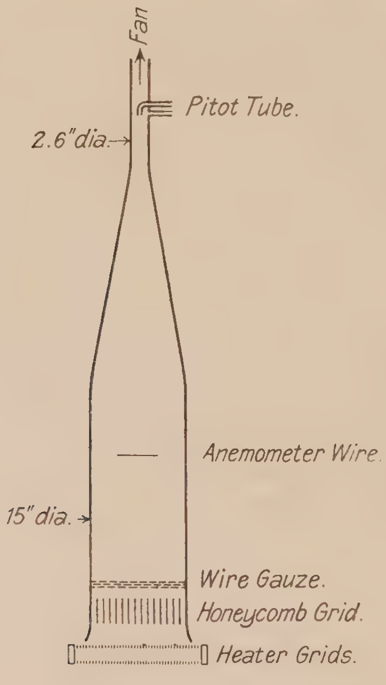

FIG. 2.—VERTICAL HOT-AIR CHANNEL.

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* Morris, " Electrician," October 4th, p. 1056, 1912.

† Since this Paper was first written, Morris (" Electrician," August 27, 1920) has described an instrument for measuring air velocity in mines, in which the conversion of mass velocity to actual velocity is effected for each temperature by appropriate setting of the current. The present Paper shows that such adjustment is needed even to get mass velocities.

remains, retaining its other functions, altering the current in the bridge and the sensitivity of the galvanometer. A simple key enables one millivoltmeter to be used either as A or as G, as required.

Setting the indicator at *Thermometer* sends a very small current through the wire. As before, the air temperature is read directly from the balance position of the slider on the slide wire. At the same time a second scale shows the anemometer current appropriate to the temperature.

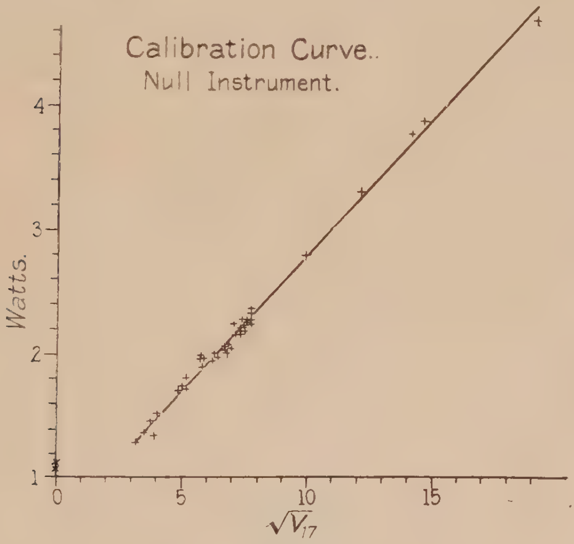


Fig. 3.

Setting the indicator at *Anemometer* sends a heating current through the wire, and adjusting this to the appropriate value, the mass velocity of the air is read directly from the galvanometer scale. The reason for this is clear from the theory of the instrument in the anemometer position.

Let P, P, R, and $(X+L_\theta)$ be respectively the resistances of the four arms of the bridge and let L_0, L_t, L_θ be respectively the resistance of the wire at 0°C., at air temperature t°C., and when heated by the current c to θ°C. above air temperature. From the thermometer setting $R=X+L_t$. Then if g be the current through the galvanometer of *high* resistance G, we have $g/c=L_0a\theta/2G$, $L_\theta=L_t+L_0a\theta$, $c^2L_\theta=\theta.f(v\rho)$,

whence $(2G/L_0a)g=L_tc^3/[f(v\rho)-L_0ac^2]$.

It will be seen that if c be chosen so that $L_t c^3$ is constant, and if $[f(v\rho)-L_0 ac^2]$ does not vary appreciably for the necessary variations of c, then the galvanometer reading is a function of the mass velocity only. The appropriate current c determined by L_t, depends on the temperature of the air stream only. For a range of 100°C., only 12 per cent. variation in c is necessary to keep $L_t c^3$ constant. The slight effect of this variation on $[f(v\rho)-L_0 ac^2]$ may be approximately allowed for by choosing the current appropriate to L_t such that the instruments reads quite correct for one standard velocity $(V_s\rho)$ in the middle of the scale. Thus, instead, of making $L_t c^3$ constant, make

$$L_t c^3/[f(V_s\rho)-L_0 ac^2]=\text{constant}.$$

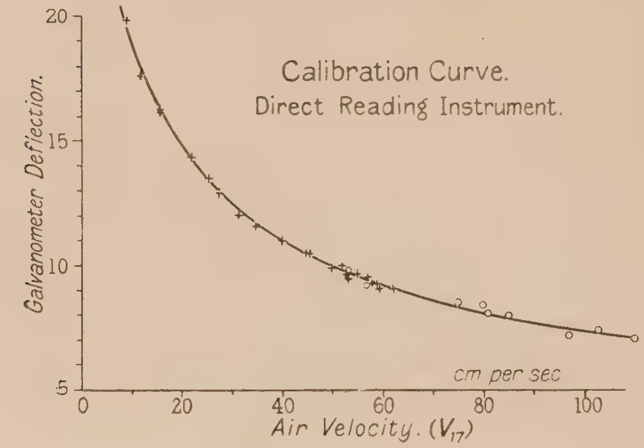


FIG. 4.

Calculations given in Table I. show for two air temperatures, 17°C. and 100°C., the appropriate currents and the galvanometer deflections produced by them for a range of air velocities.

TABLE I.

Velocity.	Galvanometer deflection.	
v_{17} cm./sec.	17°C. $c=1{\cdot}00$	100°C. $c=0{\cdot}929$
9·0	20·1	19·6
42·1	10·7	10·7
100	7·3	7·3

The anemometer current varies but little over the range of temperature and the instrument is quite satisfactory.

The actual instrument was calibrated in the wind channels by means of the previously calibrated null instrument. The results are shown graphically in Fig. 4, where it is seen that the form of the curve agrees excellently with that calculated, and that no dependence on the temperature of the air is revealed. Finally, the scale of equal divisions was removed from the galvanometer, and a direct velocity scale engraved.

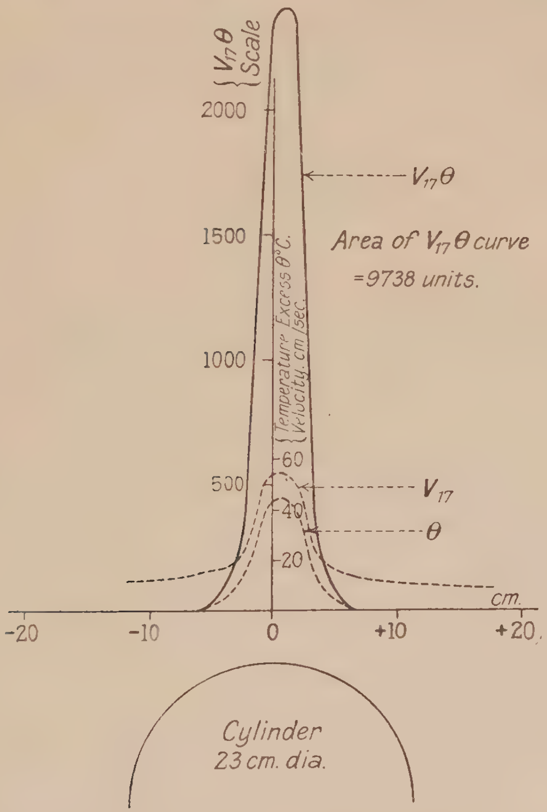


FIG. 5.—CONVECTION CURRENT ABOVE HORIZONTAL CYLINDER.

EXAMPLE OF MEASUREMENT OF CONVECTION.

The heat loss by convection from unit length of a horizontal cylinder was measured by means of the instrument and checked from a knowledge of the actual watts consumed, after allowing for radiation. Keeping the exploring wire always parallel to the axis of the cylinder, the distributions of velocity and temperature were obtained for the air passing vertically upwards through a horizontal plane about 10 cm. above the cylinder.

In Fig. 5 the curves show for the air stream the temperature excess (θ) above room temperature, the velocity (v_{17}), and the product $v_{17}\theta$. The abscissæ represent distances to the right or left of the vertical through the axis of the cylinder. Using (1*a*), the heat (h) carried off per unit length of the cylinder is readily seen to be

$$h=0{\cdot}000289\int(v_{17}\theta)dx$$

i.e.

$$=0{\cdot}000289 \quad (\text{Area of the } v_{17}\theta \text{ curve})$$

$$=2{\cdot}8 \text{ calories per cm. per sec.}$$

The value obtained by other means was 3 calories per cm. per sec., and is in good agreement, considering the sources of error associated with convection measurements using large bodies. Fuller details of the use of the instrument in this manner may be published later.

The combined instrument has been designed and developed for measuring certain convection losses and currents in elucidating the laws of convection of heat—work which has arisen out of and forms part of researches on cold storage insulation being carrried out at the National Physical Laboratory under the direction of Dr. Ezer Griffiths, for the Food Investigation Board of the Department of Scientific and Industrial Research. The author desires to thank Dr. Griffiths also for advice in the preparation for publication.

Dr. T. Barratt communicated the following remarks on the Paper by A. H. Davis :—

As Mr. Davis remarks the laws of convection are not accurately known, even in ideal cases. One of the greatest difficulties in their determination lies in the fact that the total loss of heat from a heated surface is made up ot three parts—conduction, convection, and radiation. As unfortunately there is no perfect insulator of heat, all three causes come into operation in every experiment.

In certain cases I have shown that it is possible to determine separately the losses due to convection and to radiation, the conduction loss being made inappreciable. (Proc. Phys. Soc., XXVIII., December, 1915, and XXXII., August, 1920). "Convection" may be taken as that portion of the heat emitted which is carried away by, and in the direction of the cooling fluid, while "radiation" is emitted equally in all directions from a homogeneous heated surface.

Experiments in which the convection was what is usually termed "free" or "natural" convection showed that:

(1) Out of 100 parts of "total heat" emitted from a wire 1 mm. diameter raised 10°C. or 12°C. above its surroundings at 17°C., 2·5 parts consist of radiation if the wire is uncoated, and 12·6 if it is "black." At 100°C. (the wire being again raised 10°C. or 12°C.) these percentages become 44 and 20·7 respectively. There is an increase of 20 per cent. in "total loss of heat" at the higher temperature, which is caused chiefly by increased convection.

The last sentence does not accord with Mr. Davis's remark that "there is no reason for appreciable departure from the law $H/\theta=B\sqrt{v\rho}+C$ through change of actual air temperature for temperature *excess* is the essential factor." Many measurements have been made of the rate of emission of heat from 1 sq. cm. of surface of wires, &c., for 1°C. excess. Lees (Bak. Lect., 1908) found values 0·000126, 0·000160, and 0·000200 at temperatures —155°C., —64°C. and +23°C., respectively, for rods about 6 mm. diameter. My own experiments on rods of similar dimension gave 0·000224 at 18°C., and 0·000265 at 100°C.

(2) When a cylinder is heated to 100°C., in air at about 20°C. the amount of "free" convection, *after deducting the radiation loss*, is inversely proportional to the square root of the diameter. For a sphere (within certain limits of diameter) this convection loss appears to be inversely proportional to the cube root of the diameter.

The amount of radiation from a very thin wire, such as is used in a hot-wire amemometer, must be a very small proportion indeed of the convected heat. Experiment has indicated that Newton's "First Power Law" of Cooling is very closely applicable to the *convection*, whether "free" or "forced." Where the radiation, as in very thin uncoated wires, is a very small proportion of the total heat emission, the law may safely be accepted for the total loss of heat. This has in fact been tacitly assumed in mathematical analyses in Thermal Conductivity for a century or more. When the cylinder, however, has a diameter of a millimetre or more Newton's law becomes less and less applicable, especially if the heated surface has a high absorptive power.

Results very near the truth can, however, be obtained by applying the First Power Law to the Convection loss alone, and the Fourth Power Law to the Radiation.

Mr. A. H. Davis, in reply to Dr. Barratt, writes :—

Since the theory of the hot-wire anemometer is that of forced convection and Dr. Barratt's communication is confined mainly to natural convection it is convenient to notice the difference between the two.

In free convection the hot body itself sets up the motion of the air, which depends upon gravity, and upon the relative decrease in gas density caused by 1 deg. rise in temperature. Although these variables must occur in the theoretical equation for free convection they appear to be negligible in the case of forced. An experimental difference may be noticed. Newton's first power law of cooling appears to have a fairly wide application to forced convection, compared with its application to free convection. In this latter case a $\theta^{5/4}$ law seems to be a better approximation over a wide range, as indicated by the experiments of Dulong and Petit, Peclet and Compan (thermometers, spheres, &c.). Langmuir has obtained results for wires and for a plane disc in which the law seems satisfactory up to 600°C. It is a good approximation for some larger bodies now being used in some convection experiments at the National Physical Laboratory.

It is not, of course, directly possible to predict a temperature effect with forced convection from free convection data, particularly as the density change coefficient referred to above depends markedly on temperature. The following is in amplification of remarks in the Paper on this effect. In the formula $H/\theta = B\sqrt{(v\rho)}+C$, for a given wire B and C depend only on the physical properties of the fluid of the stream, and these of course, change with temperature. A calculation was made of the probable effect of a change of 100°C. in actual temperature, and it appeared that the effect was within limits of experimental error met in using the instrument. (The calculation followed the general lines of a Paper now in the press ("Phil. Mag.") correlating data for cylinders in a stream of fluid.) The subsequent experimental work in the vertical channel with hot air up to 60°C. did not reveal that any vital factor had been overlooked. It would be necessary to introduce a correction with high temperature gases, but the complications do not appear to be great. As Dr. Barratt indicates, the radiation loss from a thin anemometer wire working at moderate temperature excess in a stream of air is practically negligible, and this fact is utilised in the design of the instrument.

The Katharometer.

1. "*The Katharometer,*" *by* G. A. SHAKESPEAR, *M.A., D.Sc.*

THE Katharometer in its most usual form consists of a cylindrical block of copper about 4 cm. long and 2·5 cm. diameter. Parallel to the axis of the block are drilled two holes (about 0·6 cm. diameter) extending from one end nearly to the other. In each of the cells thus formed is a helix of platinum wire about 0·001 inch thick. One end of each helix is in electrical contact with the block, the other end of each being connected with a lead which passes out through an insulating air-tight plug. One of the cells communicates with the outer air through three small holes drilled through the end of the block, the other cell is hermetically sealed. In another form of the instrument two other holes are also drilled parallel to the axis of the block and each of these communicates with one of the cells through two or three small apertures, the gases to be examined are passed through these subsidiary cells or pipes. This form is called a double-flow katharometer and is very useful for comparing two gases of slightly differing composition.

The two helices form two arms of a Wheatstone bridge, the other two arms usually being of manganin. These bridge resistances may either be fixed or adjustable as required, and the whole may be contained in the "head" of the katharometer. The main current in the bridge is usually 0·1 amp., which suffices to maintain the platinum helices about fifteen centigrade degrees above the temperature of the block, their heat escaping mainly by conduction through the surrounding gas. Hence a small change in conductivity of the gas in one cell (the "exposed" cell) produces a change in temperature of the wire therein and the balance of the bridge is upset. Readings may be made of the galvanometer with fixed bridge resistances or of slide wire reading with balanced bridge.

The sensitiveness of the instrument is remarkable. For example, it may easily be made to give 100 scale divisions deflexion for 1 part of hydrogen in 1,000 of air, by volume. Hence, by estimating to 0·1 scale division 1 part in a million could be detected. The capacity of the cell being about 0·5 cm.^{3} the actual volume of hydrogen used is 5×10^{-7} cm.^{3}. Of course, for such refinements of measurement, disturbances must be guarded against. During the war we applied the katharometer to a variety of uses, such as the measurement of permeability of balloon fabrics to hydrogen and other gases, the detection of leakages in balloons and the improvement of the seams where the fabric is joined, the detection of hydrogen in air and of air in hydrogen. For this last purpose a special unsymmetrical form of instrument was developed by Dr. H. A. Daynes (for reasons which will be apparent on reading his Paper in Proc. Roy. Soc., Vol. XCVII., p. 273, 1920). I have also used a katharometer in conjunction with a "thread-recorder" as a recording hygrometer for continuous records of humidity for many months—it can be made to give the pressure of water vapour in the air directly. If the comparison cell be kept saturated with water vapour (instead of dry) the instrument records the difference between saturation and existing vapour pressures, *i.e.*, the "dryness" of the air. The double-flow form of instrument is extremely useful for measuring small traces of impurity in certain cases. There are many obvious applications of the instrument, some of the more important of these will be described by Dr. H. A. Daynes. The theory of the instrument has been worked out by Dr. Daynes in his Paper "On the Theory of the Katharometer" (Proc. Roy. Soc., Vol. XCVII., p. 273, 1920). Various communications on the subject by Shakespear and Daynes will be found in the Reports of the Advisory Committee on Aeronautics, 1916-18.

2. "*Industrial Applications of the Katharometer,*" *by* Dr. H. A. Daynes (*The Cambridge & Paul Instrument Co., Ltd.*).

The katharometer described by Dr. Shakespear has a great many applications to the measurement of the composition of industrial gases, and I will confine myself to discussing this field of application. It will be as well to summarise the points which favour the use of this method of analysis in controlling manufacturing processes.

(1) The instrument gives a continuous reading, and does not need intermittent withdrawals of samples of gas and subsequent analysis.

(2) The reading of the instrument does not depend on the rate of gas flow through it, so no regulation is necessary for this.

(3) The instrument is very quick in action. The sensitive element itself has a half value period of only about 15 seconds in the case of hydrogen in air, and about 1 minute for CO_2 in air, varying for other gases according to the diffusion constant of the mixture.

(4) The zero of the instrument is reasonably independent of variations of temperature.

(5) The zero is also very nearly independent of the current flowing through the bridge.

(6) The thermal conductivity on which the operation of the instrument depends, is very nearly indeed independent of atmospheric pressure, there being very little loss of heat from the wire by convection. It is not necessary therefore to reduce the gas to normal pressure before measurement or to make any corrections.

(7) The instrument can be made quite robust; in fact, quite as robust as many industrial thermo-couples.

(8) Being ultimately an electrical measurement calibration and standardisation are easy, parts can be made completely interchangeable, and any number of sensitive elements can be connected as in distance thermometry to a single recorder or indicator.

Against these advantages we have to set three disadvantages :—

(1) The chief limitation of the method is that it can give no qualitative analysis, and can only give quantitative analysis in

certain special cases. Very complicated mixtures are almost impossible to deal with.

(2) The sensitiveness depends slightly on temperature. This is inseparable from the nature of the instrument, because the thermal conductivity increases with temperature at the rate of about one-fifth of 1 per cent. per 1°C.

(3) The sensitiveness depends very largely on the current in the bridge, being nearly proportional to the cube of the current. Adjustment is only necessary about once a day.

The chief field for the method is in cases of binary mixtures of known constituents. The graph will show the relative sensitiveness to various gases mixed with air. The deflections are those given on a double pivoted galvanometer of robust construction as supplied for industrial pyrometry, which gives about 30 deg. per millivolt. It will be seen that hydrogen is the most effective, as might be expected. The katharometer has been used for detecting dangerous mixtures in the neighbourhood of hydrogen generating plants and in airship sheds. It has also been used in research on the leakage of hydrogen and helium through balloon fabrics, and for the detection of leakage from inflated airships and balloons; also for the measurement of the purity of hydrogen in hydrogen generators and in the envelopes of balloons and airships. The measurement of hydrogen and nitrogen in mixtures during the synthesis of ammonia is an interesting and important case. Nitrous oxide, carbon dioxide, methane and water vapour in air are all mixtures which lend themselves readily to analysis by this method.

The analysis of ternary and complex mixtures is only possible in certain cases. The analysis of producer gas, blue water gas, coke oven and blast furnace gases has not yet been attempted for this reason, but it is hoped to deal with these in time.

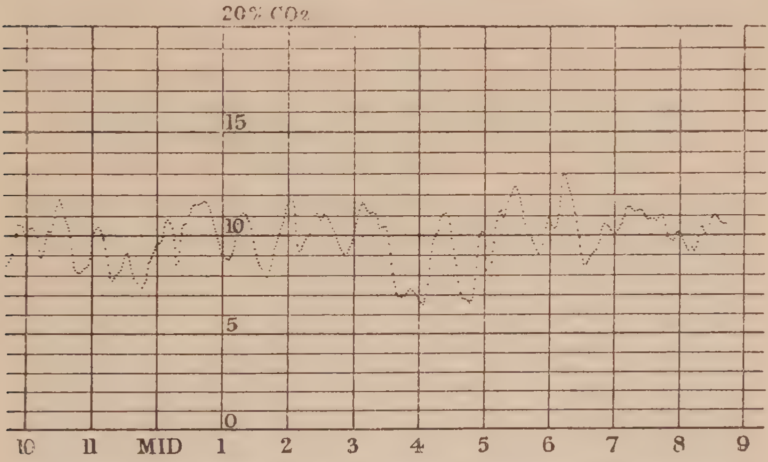
(1) When the percentages of all other gases but the one to be measured are small and the gas to be measured can be readily absorbed without affecting the others, the double flow method is used, in which both sides are exposed to the gas, one before and one after absorption of the gas to be measured.

(2) The effect of water vapour can always be counteracted automatically. If the gas is known to be saturated the other platinum arm can also be exposed to saturated water vapour, and in any case it is quite easy to saturate the gas being

measured, so that this compensation is always possible. It is much easier than drying the gas before measurement.

(3) When all the other constituents but the one of interest are very nearly equal to one another in thermal conductivity there is no difficulty.

All these three cases are illustrated in the one which promises to be the most important of all, and which has already been developed for industrial use—that is, the measurement of CO_2 in flue gases. The gases present are nitrogen, oxygen, carbon monoxide, carbon dioxide, hydrogen, methane and water vapour. Of these, nitrogen, oxygen and carbon monoxide have very nearly the same conductivity. The residual of hydrogen and methane in flue gases is extremely small, and the gas is always saturated at the temperature of the instrument, so that water vapour can easily be compensated for. Another illustration of a complex mixture is that of water vapour in blast furnace gases. Here there are other gases present, 3 or 4 per cent. of hydrogen and 10 or 12 per cent. of CO_2. These in themselves would give very much larger deflections than would be given by the water vapour, but the effect of these gases is eliminated by exposing both platinum coils to the gas, the only difference being that on one side the gas has to diffuse through a chamber containing phosphorus pent-oxide to absorb the water vapour.



REPRODUCTION OF RECORD FROM CAMBRIDGE ELECTRICAL CO_2 RECORDER.

For measuring the percentage of CO_2 in boiler flue gases, the katharometer is mounted in a head which carries the two manganin bridge coils and a series coil in the galvanometer circuit which makes all the katharometers give the same E.M.F. for a given current in the bridge, and so makes them completely interchangeable, the galvanometers where recorders or indicators are also standardised to the same resistance and sensitiveness. This makes the addition of extra points on to an existing outfit very simple. The bridge coils of the katharometer being entirely in the head, the leads do not come into the arms of the bridge, so that it is not necessary for the leads to be balanced. Four-way leads are taken from the katharometer to the indicator or recorder wherever it may be placed, distance is no difficulty at all. The katharometer is screwed directly into a ¾ in. gas pipe leading from the back of the boiler or from any other point, and the gas is drawn through this pipe by a simple water aspirator. In plants where there is sufficient difference of pressure between the boiler and the stack, it is possible to use this difference of pressure to produce the necessary flow of gas without the use of an aspirator. Water vapour is compensated for by a small brass box filled with water, and screwed on to the side of the katharometer into a hole which indicates with the platinum spiral not exposed to the gas.

The gas has, of course, to be cleaned before being drawn down the pipes and past the katharometer. This is done by a separate soot filter. The final protection of the katharometer against deposit of soot is very simple because the gas does not have to flow actually into the instrument, but only past it, so it is possible to put a glass wool filter in between the katharometer and the pipes through which the gas is flowing without causing any appreciable lag due to the diffusion of the gas from the pipe to the katharometer.

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"Further Applications of the Katharometer." G. A. Shakespear and H. A. Daynes.

PROCEEDINGS OF THE ROYAL SOCIETY, A. VOL. XCVII., 1920.

"The Process of Diffusion through a Rubber Membrane." H. A. Daynes, M.Sc.

"The Theory of the Katharometer." H. A. Daynes, M.Sc. (with an introductory note by G. A. Shakespear).

Caleometer.

"The Caleometer," by PROF. LEONARD HILL, *F.R.S., and* MISS HARGOOD-ASH.

THE Caleometer designed by the late O. W. Griffith and Leonard Hill is an instrument which measures the heat loss of a wire coil maintained at a constant temperature.

The arrangement of the instrument is shown diagrammatically in Fig. 1. *A B C D* form the arms of a Wheatstone bridge. The arm *AD* contains the coil which is made of a resistance wire having a large temperature coefficient. The bridge is balanced when the coil is at a temperature of 34°C. and the galvanometer *G* then shows no deflection. If the coil changes temperature the balance is upset, the galvanometer needle is deflected, and makes contact with *H* by means of one of the two platinum pointers *E* and *F*. This closes either the circuit *QHELMPS* or *QHFNMPS*, in the former case, which occurs when the temperature of the resistance coil has fallen, the soft iron plunger, attached to the movable contact *K* by a string passing over a pulley wheel, moves into the part of the coil *LM* and the resistance *R* in the battery circuit is reduced, and the current in the bridge increased, in the latter case which occurs when the temperature of the resistance coil rises the plunger moves into the part of the coil *NM* and the resistance at *R* is increased, the current in the bridge being decreased. In either case the temperature of the coil is brought back to 34° C. In practice to prevent sticking of the contacts it is found advisable to have *H* revolving.

The ammeter *W* in series with the resistance coil shows the amount of current required to flow through the coil to keep it at 34°C., and enables the heat loss of the wire to be calculated. The instrument may be made self recording, when the position of the ammeter needle is marked, at regular short intervals of time, on a piece of paper drawn at a known speed beneath it. The records from two coils may be obtained in this way for the same period, the coils being in different atmospheric conditions.

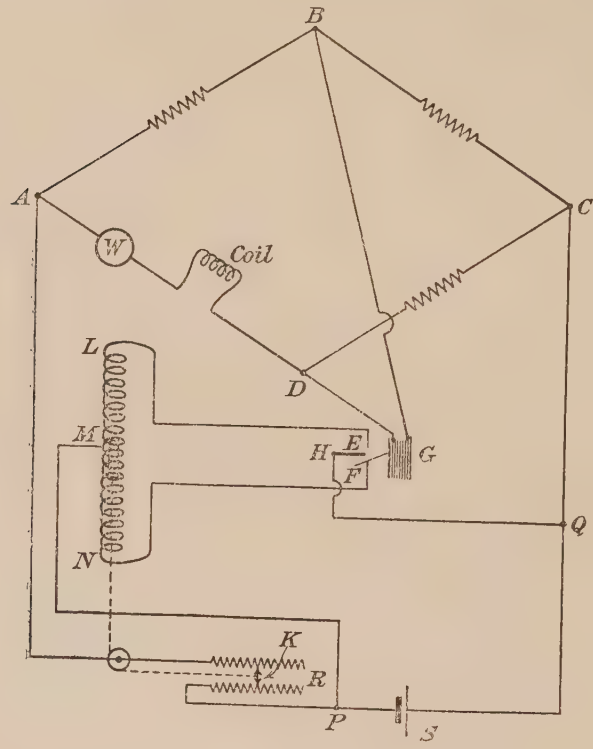

FIG. 1.—THE CALEOMETER. DIAGRAM OF CONNECTIONS.

The heat loss from the coil is affected by air currents, also, owing to its low temperature, by changes of temperature. The Caleometer, therefore, enables the meteorologist or hygienist to record daily variations of the rate of cooling of a surface at body temperature produced by atmospheric conditions. The records taken out of doors thus obtained throw light on the causation of those feelings which lead us to define the weather as pleasant, unpleasant, bracing, muggy, &c. Records taken indoors enable one to measure the cooling power of the atmosphere on a surface at body temperature and guide one to the securing of adequate ventilation. The Caleometer acts as a recording kata-thermometer the use of which for measuring the adequacy of ventilation has become established. The resistance coil is then placed within the bulb of a kata-thermometer. The instrument is made by the Cambridge and Paul Instrument Co., Newton Avenue Works, New Southgate, London, N.

Liquid Depth Gauge.

"*Liquid Depth Gauge, Distant Reading Type,*" *by* EDGAR A. GRIFFITHS.

IT is a well-known fact that the cooling power of a liquid is vastly greater than that of a gas, and this was taken advantage of by the author in the design of a gauge for measuring the depth of petrol in the tanks of aeroplanes. The gauges usually employed for this purpose are of the float type, sometimes involving transmission to a dial by the use of a string. In practice such gauges are not very satisfactory, particularly so if the tank is under air pressure. In the present instrument the gauge has no moving parts in the tank.

It consists essentially of a thin wire of platinum electrically heated to a temperature excess of about 20 deg. above the surrounding air. The wire is insulated and suitably protected by a tube projecting to the full depth of the tank. The portion of the wire immersed in the liquid is cooled down to practically the same temperature as the liquid, while the part above the surface is at the excess temperature.

Since the temperature coefficient of the resistance of platinum is of the order of 0·4 per cent. per degree, it follows that the average temperature and hence the resistance of the wire, will depend upon its depth of immersion in the liquid. Now the most convenient method of measuring changes of electrical resistance is by means of the Wheatstone bridge, and in this gauge the wire forms one arm of the bridge as shown in the diagram (Fig. 1). The influence of changes in the temperatures of the liquid and the atmosphere above are eliminated by arranging alongside a similar wire, totally sealed off from the liquid and electrically connected in the other arm of the bridge.

The changes of the resistance of the partially immersed wire, with variations of liquid level, is indicated by the deflection of the galvanometer pointer. The sensitivity of

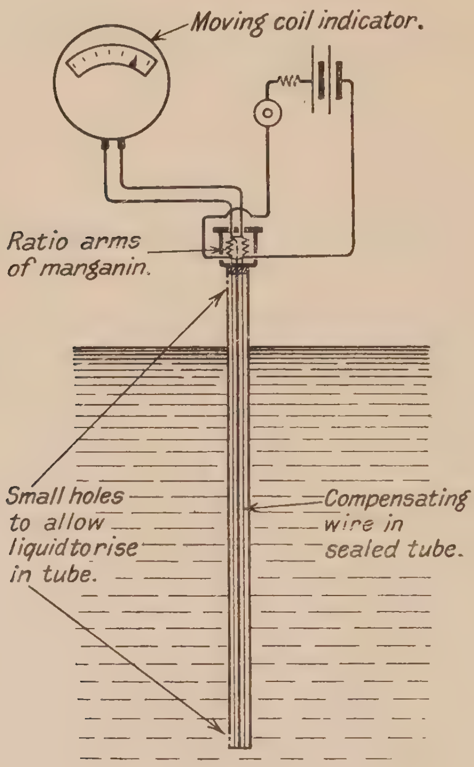
Moving coil indicator.
Ratio arms of manganin.
Small holes to allow liquid to rise in tube.
Compensating wire in sealed tube.

Fig. 1.—Liquid Depth Gauge.

the method is such that a very small elevation of temperature of the wire suffices, and in practice a robust form of moving coil instrument is employed as indicator.

Since the sensitivity of any bridge arrangement is a function of the current, it is necessary to keep this constant. The customary procedure is to have an adjustable series resistance in the battery circuit, and by means of a throw over switch, replace the wire by a dummy coil totally immersed. The current in the circuit is then adjusted to give full-scale deflection, the procedure being identical with that employed with direct reading resistance thermometers.

An alternative way would be to employ the same indicator with a shunt as ammeter when it is desired to adjust the battery current to a predetermined value.

The necessity for such periodical adjustments is a great disadvantage for aircraft purposes, when the attention of the pilot is fully occupied with more vital controls, and, consequently, the apparatus was modified to avoid this necessity. In this arrangement an iron wire ballast resistance is inserted in the battery circuit. The iron wires are sealed in a bulb similar to an electric lamp, with an atmosphere of hydrogen.

Such a resistance will maintain a fairly steady current even if the voltage of the battery changes by as much as 20 per cent., so frequent adjustment of the current is not necessary. This property of iron wire is, of course, well known and iron wire resistors will be found in most commercial types of Nernst lamps to compensate for the negative temperature coefficient of resistance of the rare earth mixture, of which the glower is composed.

As the two ratio arms of the bridge are of manganin of negligible temperature coefficient, calculation shows that a constant battery current gives a constant sensitivity for the instrument for temperature changes within ordinary limits.

XII. *On Tracing Rays through an Optical System. (Fourth Paper.) By* T. Smith, *M.A., F.Inst.P. (From the National Physical Laboratory.)*

Received October 21, 1920.

ABSTRACT.

The Paper, which is mainly mathematical, is a further development of the system described by the author in the previous Papers of the same title presented to the Society; formulæ for skew rays are put into a shape as far as possible similar to those applying to rays in one plane.

The formulæ most generally used for tracing through an optical system rays which do not lie in a plane through the axis are not arranged along lines parallel to those followed in tracing rays contained by such a plane, and hence they do not readily exhibit the character and extent of the differences to be expected between rays of these two classes. The formulæ given in this Paper are the result of an attempt to bring the treatment of the more general rays into closer conformity with that adopted in the simpler case.

To secure the desired features in the formulæ it is proposed to obtain, the equations for the simpler case should be derivable at once by substituting zero for some quantity on which the skewness of the ray depends. The obvious variable to assume which may thus vanish is the distance between the ray and the axis. Let this be denoted by Δ. The distance from the origin at which this closest approximation is attained will evidently be of importance in the problem. Let it be denoted by l, reckoned positive when the origin is to the right of the position of nearest approach. The equation to the ray may then be written in the form

$$\frac{x+l}{\cos\psi}=\frac{y-\Delta\sin\theta}{\sin\psi\cos\theta}=\frac{z+\Delta\cos\theta}{\sin\psi\sin\theta}, \quad . \quad . \quad . \quad . \quad (1)$$

so that Δ may be calculated from

$$\Delta=y\sin\theta-z\cos\theta=\frac{Ny-Mz}{(M^2+N^2)^{\frac{1}{2}}}. \quad . \quad . \quad . \quad (2)$$

and l from

$$\left.\begin{aligned} l&=(y^2+z^2-\Delta^2)^{\frac{1}{2}}\cot\psi-x=\left(\frac{y^2+z^2-\Delta^2}{M^2+N^2}\right)^{\frac{1}{2}}L-x \\ &=\frac{L(Lx+My+Nx)-x}{M^2+N^2} \end{aligned}\right\}, \quad . \quad (3)$$

where x, y, z are the co-ordinates of any point on the ray, ψ is the inclination of the ray to the axis, and L, M, N are its direction cosines. The problem for solution need not involve the actual direction and position of the ray, but on the contrary does not concern the value of θ. It is thus only necessary to know l, Δ and ψ to enable a solution to be reached. The origin of co-ordinates may be taken at the vertex of the refracting surface under consideration, and the problem is to find from these quantities and the data of the system the powers of the various surfaces for the given ray and the length of its path between successive surfaces. It will suffice to consider a single surface and to show how l', Δ', and ψ', or rather $\sin\psi'$, after refraction, may be obtained from the known values of l, Δ and $\sin\psi$, before refraction.

The determination of these quantities will be most readily followed by reference to the accompanying figure.

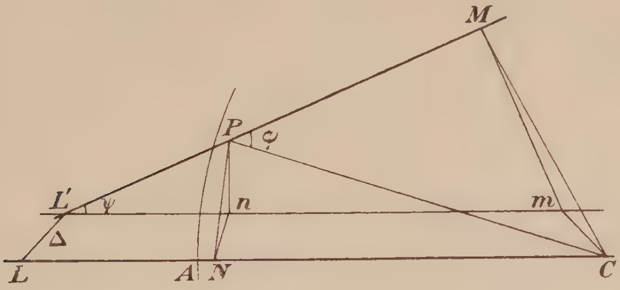

FIG. 1.

LC is the axis of the system, coinciding with the axis of x, and meeting the vertex of a refracting surface at A and its centre at C. $L'PM$ is an incident ray, LL' is the shortest distance between the ray and axis, and is perpendicular to both. P is the point of refraction, and M is the foot of the perpendicular from C to the ray. Let the rectangle $L'LCm$ be completed, and m joined to M. Then mM is perpendicular to the ray and to mC, and it at once follows that if the radius of the surface is r,

$$r^2 \sin^2 \varphi = (l+r)^2 \sin^2 \psi + \Delta^2. \quad . \quad . \quad . \quad . \quad (4)$$

The evaluation of $\sin\varphi'$ and K at once follows from

$$\mu' \sin \varphi' = \mu \sin \varphi,$$

and

$$rK = \mu' \cos \varphi' - \mu \cos \varphi,$$

equations common to other algebraic methods of calculation. Before refraction can be considered the x co-ordinate of P is required. Let $P\,N\,n$ be the plane through P normal to the axis. Then

$$L'M = LC\cos\psi,$$
$$= L'P + PM,$$

or

$$(l+x)\sec\psi = (l+r)\cos\psi - r\cos\varphi. \quad . \quad . \quad . \quad (5)$$

The analytical equations of refraction may be written in the form

$$\mu'\cos\psi' = \mu\cos\psi + (r-x)K \quad . \quad . \quad . \quad . \quad (6)$$

$$\mu'\sin\psi'\cos\theta' = \mu\sin\psi\cos\theta - \{\Delta\sin\theta + (l+x)\tan\psi\cos\theta\}K,$$
$$\mu'\sin\psi'\sin\theta' = \mu\sin\psi\sin\theta - \{-\Delta\cos\theta + (l+x)\tan\psi\sin\theta\}K,$$

and from the two latter it at once follows that

$$\mu'^2\sin^2\psi' = \{\mu - (l+x)\sec\psi K\}^2\sin^2\psi + \Delta^2K^2. \quad . \quad . \quad (7)$$

Again, these two equations may be written in the familiar form

$$\frac{\mu'M' - \mu M}{y} = \frac{\mu'N' - \mu N}{z},$$

or

$$\mu'(N'y - M'z) = \mu(Ny - Mz),$$

showing from (2) that

$$\mu\sin\psi\Delta$$

is invariant on refraction, and indeed is constant for all refractions in a coaxial system. Thus, Δ' is determined from

$$\mu'^2\sin^2\psi'\Delta'^2 = \mu^2\sin^2\psi\Delta^2 = C^2, \quad . \quad . \quad . \quad . \quad (8)$$

this form being adopted since Δ always appears in the equations squared, as is to be expected since a change in its sign has no influence on the problem considered. It now remains to find l'. This is derived at once from the consideration of the two values of y^2+z^2 obtained from the co-ordinates of the incident and refracted rays. The equation is

$$(l'+x)^2\tan^2\psi' = (l+x)^2\tan^2\psi + \Delta^2 - \Delta'^2, \quad . \quad . \quad . \quad (9)$$

where $(l'+x)\tan\psi'$ usually has the same sign as $(l+x)\tan\psi$. Any ambiguity in sign may be avoided by using the alternative equation

$$\mu'\left\{\frac{x + l'\sin^2\psi'}{\cos\psi'} + x\cos\varphi'\right\} = \mu\left\{\frac{x + l\sin^2\psi}{\cos\psi} + x\cos\varphi\right\}.$$

The value of l for the next refraction is, of course, found by increasing the l' after refraction at the present surface by t, the axial separation of the surfaces. The ray separation of the surfaces is obtainable from a number of equations, of which

$$d \cos \psi = t - x + x',$$

where x' denotes the x of the following surface, will be as convenient as any.

The equations given above afford a complete solution of the problem. One additional equation—viz., (8)—is necessarily introduced on account of the additional degree of freedom possessed by the ray, and in place of the simple angular relation $\psi' = \psi + \varphi' - \varphi$ ordinarily used two other equations have had to be substituted. This change, however, is no drawback, and the two equations will take no longer to apply to a ray than the single equation for the simpler case. The outstanding difference in the treatment required is the extraction of three square roots—those for $\cos \varphi$, $\cos \varphi'$ and $l' + x$. These present no difficulty where a calculating machine is employed, and in such cases the new equations are not likely to be found inferior to those given previously.

Although the preceding equations form a complete solution to the problem they are not in the best form for calculation where it is desired to apply the same equations to all surfaces whether their curvatures be large or small. This fault, however, is readily remedied. For equation (4) may be rewritten as

$$\sin^2 \varphi' = (1 + lR)^2 \sin^2 \psi + \Delta^2 R^2, \quad . \quad . \quad . \quad . \quad (10)$$

and equation (5) may be combined with (4) to yield

$$\frac{l + x}{\cos \psi} = \frac{2l + (l^2 + \Delta^2) R}{\cos \varphi + (1 + lR) \cos \psi}. \quad . \quad . \quad . \quad . \quad (11)$$

The remaining equations, apart from (6), which is an additional equation, and may be written

$$\mu'(\cos \psi' - \cos \varphi') = \mu(\cos \psi - \cos \varphi) - xK,$$

require no modification. It is to be noted that l and $\sin \psi$ are necessarily always finite, and no special cases arise for separate consideration. It may notwithstanding this be convenient to regard $l \sin \psi$ as a separate variable in place of l, with a view to increasing the accuracy obtainable with a given number of figures. Let this product be denoted by λ. The modified equations then become

$$\left.\begin{aligned} \lambda&=(y^2+z^2-\Delta^2)^{\frac{1}{2}}\cos\psi-x\sin\psi \\ \text{or}\qquad \lambda\sin\psi&=L(Lx+Ny+Nz)-x \end{aligned}\right\}, \quad . \quad . \quad . \quad (3)$$

$$\sin^2\varphi=(\sin\psi+\lambda R)^2+\Delta^2R^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$\frac{x+\lambda\sin\psi}{\cos\psi}=\frac{2\lambda\sin\psi+(\lambda^2+\Delta^2)R}{\cos\psi+\cos\varphi}=r(\cos\psi-\cos\varphi)\ . \quad (5)$$

$$\mu'^2\sin^2\psi'=\left(\mu\sin\psi-\frac{\lambda+x\sin\psi}{\cos\psi}K\right)^2+\Delta^2K^2 \quad . \quad . \quad (7)$$

$$\left(\frac{\lambda'+x\sin\psi'}{\cos\psi'}\right)^2=\left(\frac{\lambda+x\sin\psi}{\cos\psi}\right)^2+\Delta^2-\Delta'^2, \quad . \quad . \quad (9)$$

and the λ for the new surface is obtained by adding $t\sin\psi'$ to this value of λ'. A check on the accuracy of the calculation is afforded by the relation

$$\sin^2\varphi'=(\sin\psi'+\lambda'R)^2+\Delta'^2R^2,$$

and others may be constructed if desired. For example, it readily follows from (7) and (8) that

$$\begin{aligned}\mu'\sin\psi'\left(\mu'\sin\psi'+\frac{\lambda'+x\sin\psi'}{\cos\psi'}K\right)&=\{(\mu\mu'\sin\psi\sin\psi')^2-C^2K^2\}^{\frac{1}{2}}\\ &=\mu\sin\psi\left(\mu\sin\psi-\frac{\lambda+x\sin\psi}{\cos\psi}K\right),\end{aligned}$$

and equation (6) may have the form

$$\mu'\left(\frac{x+\lambda'\sin\psi'}{\cos\psi'}+x\cos\varphi'\right)=\mu\left(\frac{x+\lambda\sin\psi}{\cos\psi}+x\cos\varphi\right).$$

It may be noted that the extraction of square roots may, if desired, be avoided, since the equations are in a form suitable for rapid approximation by the method of continued fractions.

XIII.—*On the Refractometry of Prisms.*

By J. GUILD, *A.R.C.Sc., F.Inst.P., F.R.A.S.* (*From the National Physical Laboratory.*)

RECEIVED NOVEMBER 26, 1920.

ABSTRACT.

In this Paper a generalised formula for the refraction of light through a prism is obtained, and the particular cases pertaining to practical methods of refractometry deduced from it. The sensitivity of various methods for various prism angles and refractive indices is shown in a series of curves, as is also the liability to error due to errors in auxiliary constants.

IN a previous Paper* the writer developed various formulæ in connection with the refraction of light through prisms. As the interest of that Paper centred round the Pulfrich refractometer, the treatment dealt only with the case of grazing incidence at the first surface of the system.

In view of their importance in connection with the design of instruments for refractive index determinations it may be worth while drawing attention to the equivalent relationships in other cases with a view to determining the comparative accuracy of different methods of refractometry, and the actual accuracy likely to be obtained with a given instrumental equipment.

Let *AB*, Fig. 1, be a ray of monochromatic light incident on the first of two plane surfaces which separate three regions in which the refractive indices for the light concerned are μ_1, μ_2 and μ_3.

We need only consider the case in which the incident ray and the normals to the surfaces all lie in the plane of the paper.

In developing prism formulæ it is desirable, as in lens formulæ, to follow a definite convention with regard to the signs of angles. Otherwise confusion and ambiguity frequently arise. The convention which the author himself finds most convenient—although it has certain disadvantages—is to regard all angles as positive when measured counterclockwise from their appropriate reference directions. For angles of incidence and refraction the reference directions are the normals to the respective surfaces: for the angles between surfaces, the normal to the surface first encountered by the ray is the reference direction for the normal to the second surface.

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* Proc. Phys. Soc., Vol. XXX., Part III., p. 157.

Figure 1 is drawn in such a way that all the angles with which we are concerned are positive on the above convention. The resulting formulæ will differ in the signs of some of the quantities from those deduced in the textbooks where only the magnitude of the angles are taken into account.

Let the angle of incidence $A\hat{B}N_1=i$. The refracted ray BC in the second medium makes an angle $N'_1\hat{B}C,=r$, with the normal at B, and finally enters the third medium, making an angle of emergence $N'_2\hat{C}E,=e$, with the normal to the second surface at C. If θ is the angle between the surfaces, it follows from the figure that $B\hat{C}N_2=r-\theta$, and that $C\hat{D}O=\theta+e-i$. But $C\hat{D}O$ is the angle by which the ray has been deviated by the two refractions $=\delta$, say.

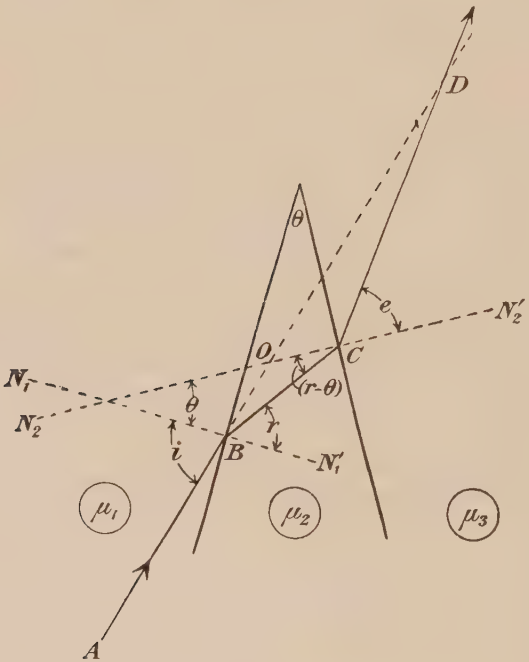

FIG 1.

$\therefore$ the deviation, δ, $=\theta+e-i$.

$$\text{Sin}\ {}^2\theta=\{\sin r\cos(r-\theta)-\cos r\sin(r-\theta)\}^2$$
$$=\sin^2 r+\sin^2(r-\theta)-2\sin r\sin(r-\theta)\cos\theta.$$

From the law of refraction

$$\mu_1\sin i=\mu_2\sin r\,;\quad \mu_2\sin(r-\theta)=\mu_3\sin e.$$

Therefore

$$\mu_2^2\sin^2\theta=\mu_1^2\sin^2 i+\mu^2_3\sin^2 e-2\mu_1\mu_3\sin i\sin e\cos\theta \qquad (1)$$

Equation (1) is the starting point from which all special cases may be deduced.

Prism in Air.

Refractive index from deviation and angle of incidence or emergence.—In the case of a prism surrounded by air $\mu_1=\mu_3=1$ (approx.) and equation (1) becomes

$$\mu_2^2\sin^2\theta=\sin^2 i+\sin^2 e-2\sin i\sin e\cos\theta \quad .\ .\ .\ .\ .\ (2)$$

which is the general equation for this case.

To determine μ_2, the index of the prism, it is necessary to know both i and e in addition to the angle, θ, of the prism. Alternatively either i or e and the deviation, δ, which the light suffers in passing through the prism will give μ_2 since $\delta=\theta+e-i$.

Angle of incidence for a given deviation.—Before departing from the general case we may treat the converse problem which occasionally arises in practice, viz., given a prism of a certain index and refracting angle to choose the angle of incidence which will give a specified deviation.

From equation (2)

$$\mu_2^2\sin^2\theta=\tfrac{1}{2}\{2-(\cos 2e+\cos 2i)\}-\cos\theta\{\cos(e-i)-\cos(e+i)\}$$
$$=\tfrac{1}{2}[2-2\{\cos(e+i)\cos(e-i)\}]-\cos\theta\cos(e-i)$$
$$+\cos\theta\cos(e+i)$$
$$=1-\cos(e+i)\{\cos(e-i)-\cos\theta\}-\cos\theta\cos(e-i).$$

But $\qquad e+i=\delta+2i-\theta$ and $e-i=\delta-\theta$;

$$\therefore\quad \mu_2^2\sin^2\theta-1+\cos\theta\cos(\delta-\theta)$$
$$=-\cos(\delta-\theta+2i)\{\cos(\delta-\theta)-\cos\theta\},$$

or

$$\cos(\delta-\theta+2i)=\frac{1-\mu_2^2\sin^2\theta-\cos\theta\cos(\delta-\theta)}{\cos(\delta-\theta)-\cos\theta}.$$

Since δ is always—ve, provided $\mu_2>1$, we may put $\delta=-\bar{\delta}$, where $\bar{\delta}$ is the numerical value of the deviation, and

$$\cos(2i-\theta-\bar{\delta})=\frac{1-\mu_2^2\sin^2\theta-\cos\theta\cos(\theta+\bar{\delta})}{\cos(\theta+\bar{\delta})-\cos\theta}, \quad . \quad . \quad (3)$$

from which, since θ and μ_2 are supposed known, the value of i corresponding to the required value of $\bar{\delta}$ can be calculated.

Minimum Deviation. It is comparatively rarely that the general case is utilised in refractometric work. It simplifies experimental arrangements to make one of the variables i, e, or δ conform to some easily attained condition so that only one quantity remains to be measured. The most familiar condition is that the deviation δ should be a minimum. This happens when the angles of incidence and emergence are equal.* In this case $i=-e$ and equation (2) reduces to

$$\mu_2^2\sin^2\theta=2\sin^2 i(1+\cos\theta),$$

or,

$$4\mu_2^2\sin^2\theta/2\cos^2\theta/2-4\sin^2 i\cos^2\theta/2,$$

whence

$$\mu_2=\frac{\sin i}{\sin\theta/2}.$$

The deviation

$$\delta=\theta+e-i=\theta-2i, \text{ or } i=\tfrac{1}{2}(\theta-\delta).$$

$$\therefore \qquad \mu_2=\frac{\sin\frac{1}{2}(\theta-\delta)}{\sin\theta/2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

GRAZING INCIDENCE.

Another convenient experimental arrangement is that in which $i=90$ deg. In this case the ray is refracted at the so-called critical angle, and equation (2) reduces to

$$\mu_2^2=1+\left(\frac{\cos\theta-\sin e}{\sin\theta}\right)^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

NORMAL EMERGENCE.

In the Abbé Autocollimating spectrometer the light is reflected normally from the second face of a 30 deg. prism. This is clearly equivalent to putting $e=o$ in equation (2), which then becomes

$$\mu_2=\sin i/\sin\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The reader may have observed that the signs of δ in equation (4) and of $\sin e$ in equation (5) differ from the usually given formulæ. This is the only disadvantage in the convention with respect to signs followed in obtaining the general equation. The fault, however, really lies in the ordinary formulæ in which the angles are assumed to have magnitude only. No confusion can arise in the case of minimum deviation for δ is invariably negative, and so the + sign obtained by neglecting signs of angles always holds. In the case of grazing incidence, however, the formula is usually developed from a figure in which e is $-ve$, and, consequently, a + sign is obtained by neglecting this fact. In practice e may quite frequently be positive and the $-$ sign is required. There are thus two formulæ to be remembered, and confusion easily arises in particular cases. By following one sign convention which holds for every case such ambiguity is prevented.

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* See any elementary textbook.

PRISM WITH ONE SURFACE IN CONTACT WITH ANOTHER REFRACTIVE SUBSTANCE.

In this case, which applies to most methods of commercial refractometry, $\mu_1 \neq 1$, and equation (1) becomes

$$\mu_2^2 \sin^2\theta = \sin^2 e + \mu_1^2 \sin^2 i - 2\mu_1 \sin i \sin e \cos\theta.$$

The only case of practical importance is that of grazing incidence, or $i=90$ deg., in which case the equation reduces to

$$\mu_1 = \pm \sin\theta\sqrt{\mu_2^2 - \sin^2 e} + \sin e \cos\theta.$$

The index μ_1 is $+ve$, and greater than unity, whereas the product $\sin e \cos\theta$ is necessarily less than unity, so that the positive sign before the square root must always be taken.

Thus $$\mu_1 = \sin\theta\sqrt{\mu_2^2 - \sin^2 e} + \sin e \cos\theta. \quad . \quad . \quad . \quad (7)$$

If μ_2 and θ are known, μ_1 can be determined from equation (7), the only measurement required being e, the angle of emergence which the critical ray makes with the normal to the second face of the prism.

SENSITIVITY OF VARIOUS METHODS.

The foregoing equations, which connect the various essential quantities in the most usual methods of refractometry, do not give any information concerning the sensitivities of the various methods, that is to say, the change in the measured quantity per unit change in the refractive index, nor do they indicate the extent to which the results are affected by errors in the values of the auxiliary constants, such as the angle of the prism, or, in the case to which equation (7) refers, the refractive index, μ_2, of the auxiliary prism.

Taking, first, the most familiar method, the quantity measured is δ, the minimum deviation. The sensitivity S may be measured by the number of seconds which δ varies when the refractive index of the prism changes by one unit in the fifth decimal place.

Thus
$$S=\frac{\partial\delta}{\partial\mu_2}\times\frac{57{\cdot}3\times60\times60}{10^5}=2{\cdot}06\,\frac{\partial\delta}{\partial\mu_2},$$
$$\mu_2=\frac{\sin\frac{1}{2}(\theta-\delta)}{\sin\theta/2},$$
whence
$$\frac{\partial\mu_2}{\partial\delta}=-\tfrac{1}{2}\,\frac{\cos\frac{1}{2}(\theta-\delta)}{\sin\theta/2},$$
or,
$$S=-\frac{4{\cdot}12\sin\theta/2}{\cos\frac{1}{2}(\theta-\delta)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Thus, the sensitivity is a function of the deviation and of the angle of the prism. We can calculate its value for any values of μ_2 and θ, first finding the appropriate value of δ from equation (4).

For the method of grazing incidence, the similar formula as shown in the previous Paper,* is
$$S=-\frac{2{\cdot}06\mu_2\sin\theta}{\cos e\sqrt{\mu_2{}^2-1}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

For the method of autocollimation by normal reflection from the second face of the prism
$$\mu_2=\frac{\sin i}{\sin\theta}.$$

The quantity measured is i, the angle of incidence for which the beam is exactly returned on its path, so the sensitivity
$$=2{\cdot}06\frac{\partial i}{\partial\mu_2},$$
or
$$S=2{\cdot}06\,\frac{\sin\theta}{\cos i}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The method is usually employed with a prism of 30 deg. refracting angle; by comparing equations (10) and (8) we see that the sensitivity with the autocollimating method using such a prism (θ=30 deg.) is always just half the sensitivity of the minimum deviation method using a 60 deg. prism.

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* *Loc. cit.*, page 179, eqn. (7).

For a given material (μ_2=constant) it is useful to observe how the sensitivity depends on the angle to which the prism is cut. In Fig. 2 this is plotted for two values of the refractive index (1·6 and 1·75) for the minimum deviation method.

The sensitivity becomes infinite for the maximum angle which the prism may have and still transmit light. In this case both the angles of incidence and emergence are grazing, and the deviation $=\theta-180$ deg. If $i=-e=90$ deg. and $\delta=\theta-180$ deg. equation (4) reduces to

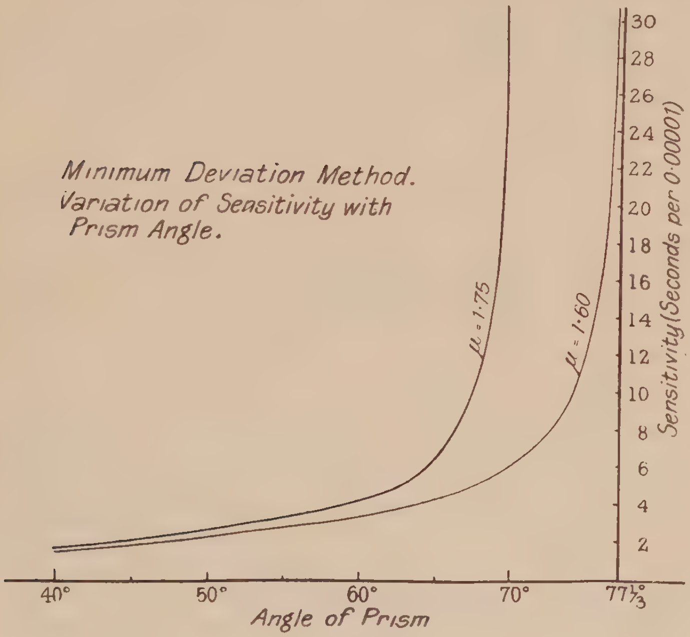

FIG. 2.

$$\mu_2=\frac{1}{\sin\theta/2},$$

which gives the largest refractive index that can be determined with a prism of angle θ or alternatively the largest angle which a prism of index μ_2 may have and still transmit light. Thus, if $\mu_2=1{\cdot}6$ the largest possible angle is 77⅓ deg., while for the usual 60 deg. prism the largest possible index is 2.

The curves for the grazing incidence method are similar in character. They are given in the previous Paper* and need not be repeated here.

From Fig. 2 it is evident that for $\mu_2=1{\cdot}6$, which is about the average of the values encountered in practical work of high precision, an angle of 75 deg. would give more than five times the sensitivity of the usual 60 deg. prism. Sensitivity is, however, only one factor in the final accuracy of the results. We shall see later that there are other factors which prevent the utilisation of the high sensitivity which might be obtained by suitable choice of prism angle, and the balance of advantages does in fact lie with the 60 deg. prism.

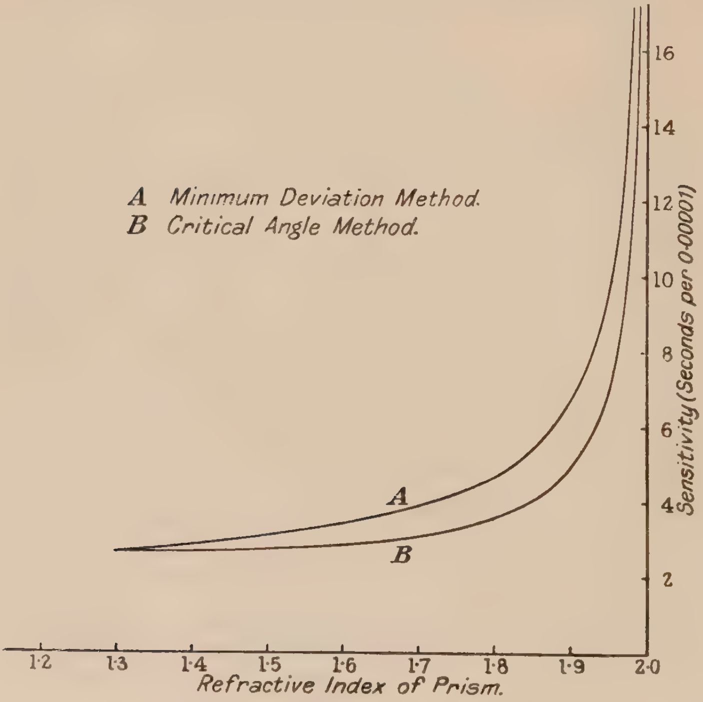

FIG. 3.

Taking this prism as basis we may compare the sensitivity of the minimum deviation and grazing incidence methods throughout a considerable range of refractive index, since these are the two methods of greatest importance in precise work.

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* *Loc. cit.*, Fig. 10.

These are plotted in Fig. 3, from which we can find the sensitivity by either method for any index. The minimum deviation method has clearly a slight advantage over the other for all ordinary materials.

EFFECTS OF ERRORS IN AUXILIARY CONSTANTS.

In the preceding formulæ the angle, θ, of the prism occurs. It is usually necessary to determine this experimentally, and its value is, therefore, subject to error. The assumption of an erroneous value for θ in calculating the index will lead to a corresponding error in the result. This is clearly proportional to the partial derivative with respect to θ of the expressions for μ_2.

For the minimum deviation method we obtain by partial differentiation of equation (4)

$$\frac{\partial\mu_2}{\partial\theta}=\frac{\frac{1}{2}\left(\cos\frac{\theta-\delta}{2}\sin\frac{\theta}{2}-\sin\frac{\theta-\delta}{2}\cos\frac{\theta}{2}\right)}{\sin^2\theta/2}$$

$$=\frac{1}{2}\,\frac{\sin\delta/2}{\sin^2\theta/2}.$$

It is better to express the result as the inverse of this, namely, the error in angle which will cause unit error in the calculated refractive index. We may conveniently term this the *tolerance in prism angle* and denote it by T_θ, introducing the factor 2·06 as before to give the result in seconds of arc per 0·00001 error in index.

Thus $$T_\theta=4{\cdot}12\,\frac{\sin^2\theta/2}{\sin\delta/2}. \qquad (11)$$

For the grazing incidence method, by differentiation with respect to θ of equation (5)

$$2\mu_2\frac{\partial\mu_2}{\partial\theta}=2\left(\frac{\cos\theta-\sin e}{\sin\theta}\right)\left\{-\frac{\sin\theta}{\sin\theta}-\frac{(\cos\theta-\sin e)}{\sin^2\theta}\cos\theta\right\}$$

$$=-2\left(\frac{\cos\theta-\sin e}{\sin\theta}\right)\left\{1+\cos\theta\,\frac{\cos\theta-\sin e}{\sin^2\theta}\right\}$$

$$=-2\left\{\sqrt{\mu_2^2-1}+(\mu_2^2-1)\cot\theta\right\}.$$

$$\therefore\quad T_\theta=2{\cdot}06\,\frac{\partial\theta}{\partial\mu_2}=\frac{-2{\cdot}06\mu_2}{\sqrt{\mu_2^2-1}+(\mu_2^2-1)\cot\theta}. \qquad (12)$$

In Fig. 4 T_θ is plotted for these two methods for different prism angles, the refractive index being taken as 1·6. Except near the limiting prism angle, for which, of course, the two methods tend to become identical, the minimum deviation method has a considerable advantage.

In Fig. 5, the tolerance with a 60 deg. prism is plotted for various values of the index.

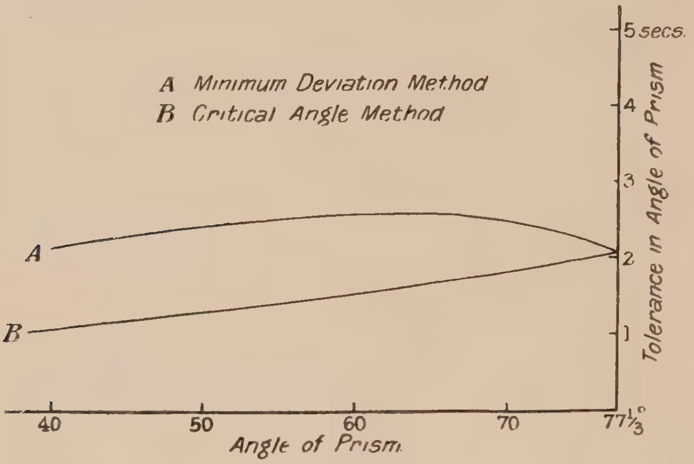

FIG. 4.

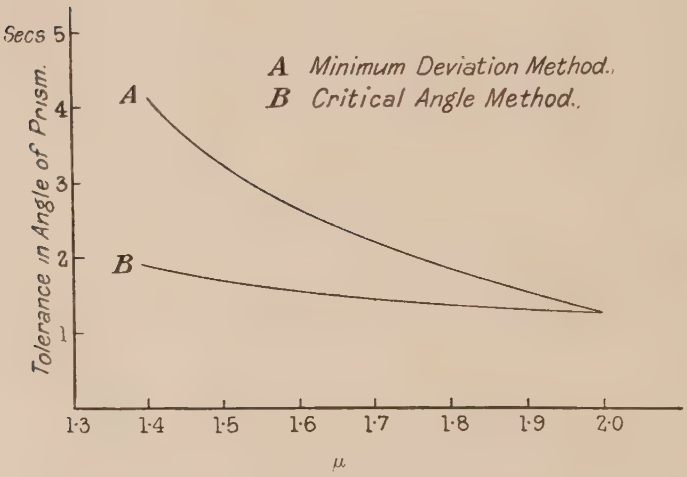

FIG. 5.

With either method we observe that the prism angle has to be known to an even greater accuracy than the deviation or angle of emergence of the refracted light. It is this that gives the 60 deg. prism the balance of advantage. For standardisation work a prism polished on all three sides can be used and each angle employed in turn, the mean being taken as 60 deg. exactly.* Where this cannot be done, on account of anisotropy, or in using a prism with only two polished faces, there is still an advantage in using a 60 deg. angle, as this can readily be determined by special methods to a high degree of accuracy. The corresponding functions for sensitivity and tolerance in angle for the method to which equation (7) refers, in which the substance under test is placed in optical contact with the first face of a prism of known properties, are fully discussed in the previous Paper, as is also the effect of error in the assumed optical constants of the standard prism. This case need not, therefore, be discussed here.

The author hopes that these two Papers, taken together will be of considerable utility to refractometrists, particularly those who have to attempt the most precise determinations, by giving in easily accessible form the means of judging at once the most useful method to employ for any particular purpose and by indicating readily the final accuracy which is possible of attainment with any particular instrumental equipment.

DISCUSSION.

Mr. T. SMITH agreed that information such as that in the Paper would help to dispel the illusions concerning the accuracy of their observations which most workers on refractive index measurement seemed to possess. He had had occasion to analyse results supposed to be the last word in accuracy, but had found quite serious errors in many of them.

He would like to see the general adoption of some system of angular notation such as that used in the Paper. The only case which presented difficulty was when applying it to a system in which there were reflecting as well as refracting surfaces. He had found it convenient in such cases to keep to the angle convention, but to change the signs of the refractive indices after reflection.

Dr. D. OWEN said the author had been speaking of tolerances of a few seconds. As a rule one felt satisfied when one got angles correct to a minute. What was the accuracy in angle which the author considered should be attained in precise measurements?

Mr. GUILD said that accuracy in angle measurements was largely a matter of choosing proper methods and apparatus. For the cases most frequently met there was no great difficulty in measuring to a second provided the surfaces of the prism were such that this accuracy had any meaning. He was at present experimenting with a very cheap and simple arrangement with which he hoped to reach a tenth of a second in cases which were fit for such precision.

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* Assuming no pyramidal error. In the case of a 60 deg. prism, if an edge of the prism is inclined to the opposite face by P seconds instead of being quite parallel to it, as in a true prism, the sum of the three angles of the prism is 180 deg. $+0{\cdot}279\ P^2\times 10^{-5}$ seconds (*vide* Guild, Trans. Opt. Soc., Vol. XXII., No. 3, 1920–21).

XIV. *A Note on the Hot Wire Inclinometer.* *By* R. H. Humphry, *M.Sc.* (*Birm.*) (*The Sir John Cass Technical Institute.*)

Received September 25, 1920.

ABSTRACT.

Two fine platinum wires were stretched parallel to each other in a hole in a copper block, and were heated electrically in a Wheatstone bridge arrangement. The changes caused by rotation were investigated with hydrogen, air and carbon dioxide surrounding the wires. It was found that both for the same current, and for the same temperature of the wires, the inclinometer filled with carbon dioxide was much more sensitive than one filled with air. The shape of the curves obtained suggests that the temperature gradient in the region traversed by the wires is nearly uniform, the difference of temperature of the layers of gas in which the wires happen to be being sensibly proportional to the vertical distance between them.

The original purpose of the experiments which are described below was two-fold. It was part of an attempt to elucidate the nature of the convection in the neighbourhood of hot wires, and also to determine if the principle could be applied to the construction of an inclinometer for use in inaccessible places where a spirit level could not be used. In view of the recently published work of J. S. G. Thomas, some of the results obtained may be of use as indicating how such an inclinometer as he describes may be made still more effective.

The apparatus consisted of two thin platinum wires, about $2{\cdot}5 \times 10^{-3}$ cm. thick, and 4 cm. long, stretched parallel to each other about 2·5 mm. apart, and symmetrically placed parallel to the axis in a hole in a copper block, the hole being about 6 mm. in diameter. The two wires formed two arms of a Wheatstone bridge, which could be balanced by moving a contact on a bridge wire. By a suitable arrangement different gases could be introduced into the cylindrical chamber containing the wires. The block could be rotated through any desired angle about a horizontal axis coinciding with that of the chamber, the zero position being that in which the wires lay in a horizontal plane.

The results for air at atmospheric pressure are shown in Fig. 1 for different bridge currents (corresponding to wire temperatures of from 60°C. to 150°C.).

For hydrogen, the effects for the same current are much smaller, while for carbon dioxide they are much greater. This is shown in the sensitivity curves of Fig. 2, and is to be expected, for the temperature of the wires would be much less in hydrogen, and greater in carbon dioxide for a given current.

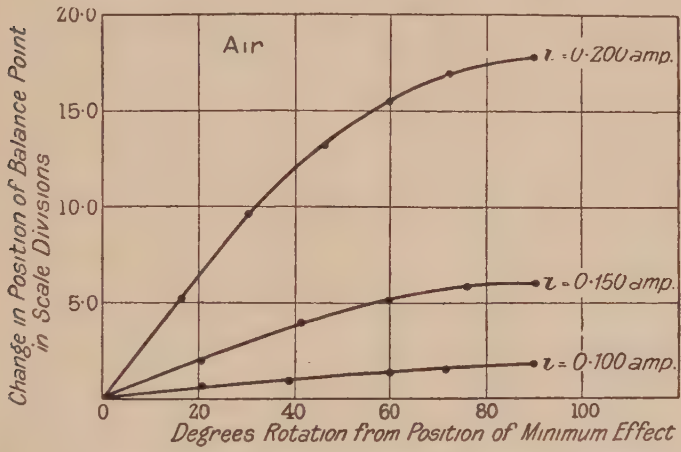

FIG. 1.—CURVES SHOWING RELATION BETWEEN CHANGE IN POSITION OF BALANCE POINT AND ANGLE OF ROTATION FOR AIR.

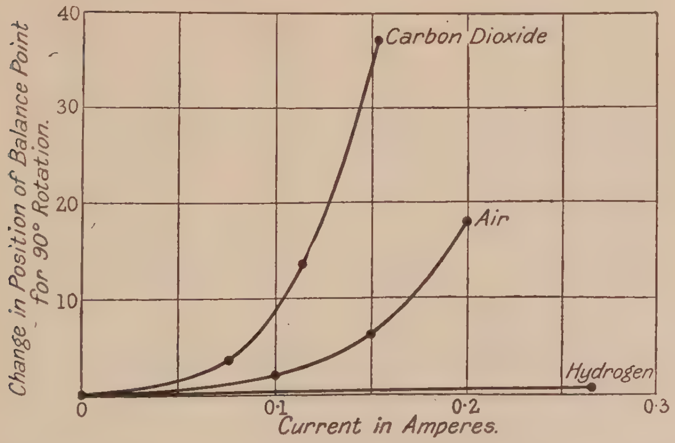

FIG. 2.—CURVES SHOWING RELATION BETWEEN CHANGE IN POSITION OF BALANCE POINT FOR 90 DEG. ROTATION AND CURRENT IN MAIN CIRCUIT FOR HYDROGEN, AIR AND CARBON DIOXIDE.

For the same temperature of the wires, the effects are also greater for carbon dioxide, and less for hydrogen as shown in

Fig. 3. This also is to be expected, for convection depends on difference of temperature, and the difference of temperature in different parts of the gas will be greater, the lower the thermal conductivity of the gas.

The shape of the curves in Figs. 1 and 3 approximates to that of a sine curve. This suggests that the gas in the chamber

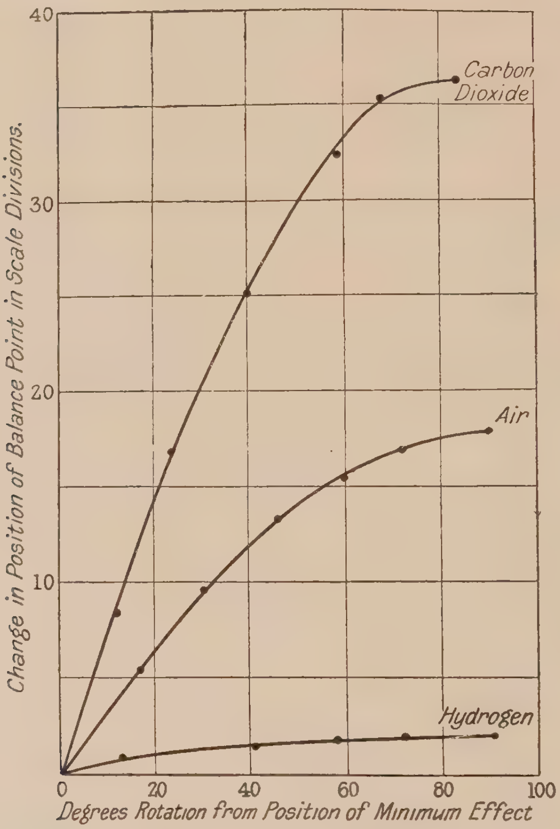

Fig. 3.—Curves showing Relation between Change in Position of Balance Point and Angle of Rotation for Same Temperature of Wire in Each Gas.

is warmer at the top than at the bottom, the gradient, at least in the region traversed by the wires, being nearly uniform, so that the difference of temperature of the two layers in which the wires find themselves is sensibly proportional to the vertical distance between them. There was no sharply defined maximum as there would have been if a narrow upward current arose from the lower wire. Probably the upward stream had lost its sharpness, by conduction, by the time when it reached the upper wire when the latter was vertically over the lower wire. The case is different from that occurring in Mr. Thomas's experiments where the distance between the wires was much smaller.

It would appear from the results here described that as an inclinometer the instrument would be much more sensitive for a given current if carbon dioxide were used instead of air. In fact (as Dr. G. A. Shakespear points out), it may be modified to serve as a katharometer for measuring the percentage of carbon dioxide, or hydrogen, or other gas mixed with air.

It may be remarked that except in special circumstances, the instrument is unnecessarily elaborate, and in practice a spirit level would in most cases be preferable. It would moreover be subject to the same disadvantages as the latter instrument in failing to give the true level if it were subjected to acceleration.

These experiments were carried out in the Physics Department of the University of Birmingham in August, 1919, under the direction of Dr. G. A. Shakespear.

Note added December 29, 1920.

Since the above was written an investigation into the temperature distribution round a heated wire has been made at the Sir John Cass Technical Institute by a method due to Mr. J. Guild, to whom the author is indebted for the suggestion. It consists in placing the hot-wire system in one of the interfering beams of a Michelson interferometer arranged to give contour fringes with sodium light.

This method is being applied to the elucidation of problems in convection and conduction in gases, and the results which have already been obtained with a hot-wire inclinometer go to confirm what has been said above.

It is hoped to be able to publish these and other results at a later date.

DISCUSSION.

Dr. J. S. G. Thomas said that the conditions under which the author's experiments were carried out made it difficult, if not impossible, to compare the results with those described in his own Paper on the subject. In his experiments the free convection loss was of much greater magnitude than the conductive loss, whereas in the present Paper the dimensions of the apparatus were such that probably conduction losses preponderated. The wires were disposed too near the walls of the containing chamber to permit convection losses to be of any great magnitude. The statement that for the same current the effects in air are less than those in carbon dioxide but greater than those in hydrogen is exactly what would be anticipated if conduction losses predominated. This result could not, however, be anticipated if convection losses predominated. Thus he had shown that, contrary to the author's contention, the temperature of a wire in carbon dioxide was not markedly different from what it was in air, provided the free convection effect was predominant. As King had shown, convection losses involved, not only the conductivity data, but likewise the specific gravity and specific heat of the medium. On the basis of King's formula, he had made a calculation of the thermal losses by convection, assuming the velocity of the convection current to be small, and had found the respective thermal losses on rotation in the case of hydrogen, air and carbon dioxide, assuming the wires initially at the same temperature, to be approximately in the ratio 12 : 100 : 200, which was not far different from the ratio of corresponding ordinates in Fig. 3. The wires in the author's experiments were so far apart that in all probability the lower wire did not experience any cooling effect when the second wire was rotated into a position at higher level. This effect itself would increase the sensitivity of the device considerably, and it was desirable to ascertain whether the sensitivity in carbon dioxide were greater than that in air when the disposition of the wires was such as to afford the maximum sensitivity in each case. It would be interesting to know whether experiments were carried out with larger currents than those shown in Fig. 2. His air results indicated that in air, with increasing current, the sensitivity increased as shown in Fig. 2, but ultimately reached a maximum value and thereafter decreased slightly. The experiments described in the previous Paper covered the range of temperature employed in the present experiments and extended to considerably higher temperatures. It would be useful to know the temperatures of the wires used in obtaining the results shown in Fig. 3, and also particulars as regards the purity or otherwise of the gases employed. In this connection, as was well known, contamination of hydrogen with minute quantities of impurities would have a marked effect upon the density of the gas, and hence of the convection loss.

Mr. J. Guild said it was of interest to him that so many workers seemed to have taken up the somewhat obvious suggestion he had made, in discussing Mr. Thomas's Paper, of investigating the temperature distribution in the neighbourhood of a hot body by means of the interferometer. He would like to give a word of caution in connection with the method, which was that the contour fringes observed would only be true isothermals if the wire were infinitely long in the direction of vision. In ordinary conditions a slight correction for end effects would have to be made in obtaining quantitative data from the fringe system.

Mr. F. E. Smith asked if experiments had been made at different pressures.

Mr. Humphry, in reply, said that it was probably true that conduction losses from the wires in his experiments were of the same magnitude as the convection losses. But the change which had been investigated must necessarily have been caused by a change in convection, since, if there were no convection at all, there would be no change in the heat losses as the wires were rotated round one another. He did not know that the ordinates in Fig. 3 were in such close agreement with those calculated from King's equation for small velocity convection currents. The region of mutual effect between the wires could easily be seen from the interferometer experiments.

The effects of rotation were not investigated with higher currents than those shown in Fig. 2, nor with temperatures higher than that in Fig. 3, except for hydrogen. The temperature of the wires in Fig. 3 was about 130°C., and with the wires surrounded by hydrogen, a higher current than 0·2 ampere was necessary to maintain this temperature. At the time when the experiments were carried out it was not thought advisable to use higher currents with air and carbon dioxide.

The gases employed were obtained from cylinders of compressed air and hydrogen, and a cylinder of liquid carbon dioxide. The purity of the hydrogen was determined by means of a katharometer, and care was taken to ensure the purity of the gas in the chamber.

Mr. Guild had said that the bands seen in the interferometer were only isothermals if the wire were infinitely long. This had been considered in connection with the experimental work which was being carried out.

In reply to Mr. Smith, experiments at different pressures had not been made with the inclinometer.

XV.—*On the Magnetic Susceptibility of Certain Natural and Artificial Oxides. By* Prof. E. F. Herroun, F.I.C., *and* Prof. E. Wilson, M.I.C.E., M.I.E.E.

Received October 6, 1920.

SUMMARY.

(1) The susceptibility of ferric oxide, as occurring in nature, varies through a wide range, but in the case of artificial preparations the range of variation may be much greater.

(2) The highest susceptibility exhibited by artificial ferric oxide occurs when either ferrous or magnetic oxide is oxidised at a high temperature in the anhydrous condition. From hydrated magnetic oxide the susceptibility, though considerable, is much less.

(3) The lowest susceptibility is obtained from native hæmatites, or naturally oxidised chalybite, or by the ignition or precipitation of ferric salts.

(4) It appears that the passage through the stage of magnetic oxide impresses more pronounced magnetic properties upon the resulting ferric oxide.

(5) The actual values of susceptibility have been obtained in the case of ferrites, one of the most conspicuous being that of copper.

(6) Heating feebly magnetic ferric oxide with a basic oxide, *e.g.*, lime or magnesia, increases susceptibility (confirming List and others).

(7) When higher susceptibility has been produced by heating ferric oxide with a basic oxide such as that of silver or mercury, subsequent removal of the metal leaves the ferric oxide in a magnetic condition.

(8) The chemical compounds (aluminates) formed when ferric oxide is replaced by aluminic oxide show no definite increase in susceptibility, which is in marked contrast with the ferrites.

Introductory.

Arising out of certain work on the magnetic susceptibility of Rock Specimens * and several varieties of Magnetite,† an endeavour has been made in the present Paper to explain certain peculiarities which were noticed in the course of those investigations. Though in general ferrous compounds, *e.g.*, Chalybite, show a higher susceptibility than the ferric oxides derived from them, certain specimens of specular iron ore were found to have a susceptibility higher than that of chalybite itself, while specimens of Red Hæmatite gave values smaller than either of the above. The view that substances having a constitution of the type of Magnetite ($FeO . Fe_2O_3$) exhibit a specially high susceptibility, appeared to merit further examination, and accordingly a large number of natural minerals and artifically prepared oxides were tested. Although several early workers at this subject prepared oxides of the above type, no determinations of susceptibility were then possible, and even later workers, who have investigated the transformation temperatures of such compounds, have not given absolute values of susceptibility. The relative importance of the FeO and Fe_2O_3 groups needed investigation, and this has been attempted in the present research. In this connection the authors regret that they have been unable to procure a specimen of Hercynite ($FeO . Al_2O_3$) sufficiently pure to test the influence of the FeO group in comparison, say, with minerals of the type of Franklinite ($ZnO . Fe_2O_3$). The number of minerals in which iron occurs in the form of FeO is very limited, Chrome Iron Ore and Pleonaste being the only examples so far tested apart from Magnetite.

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* "Phil. Trans." Roy. Soc., Series A., Vol. CCXIX. (Appendix 1919). pp. 83-87. Also "Proc." Roy. Soc., Series A., Vol. XCVI., 1920, p. 429.

† Proc. Phys. Soc., Vol. XXXI., Pt. V., 1919, p. 299.

TABLE I.

Natural Minerals.

Name.	Composition.	Density	H in C.G.S. units.	K_m in 10^{-3} C.G.S.
1.†Magnetite crystal ...	Fe_3O_4 or $FeO . Fe_2O_3$	5·06	22·5	615·0
2.†Magnetite massive .	Fe_3O_4	4·86	32	300·0
3. Franklinite	$(Zn, Mn) O . Fe_2O_3$	5·00	315	0·455
4. ,,	,,	5·01	160	0·64
5. Pleonaste	$(Mg, Fe) O . Al_2O_3$	3·80	315	0·0243
6. Automolite or Gahnite	$ZnO . Al_2O_3$	3·65	318	0·013
7. Spinel	$MgO . Al_2O_3$	3·54	765	0·00062
8. Corundum Crystal ...	Al_2O_3	3·98	756	−0·00034
9. Sapphire (blue)	,,	3·68	656	+0·00119
10. Sapphire (Siam)	,,	3·995	776	0·00143
11. Ruby (Mysore)	,,	3·96	790	0·00047
12. Red Hæmatite . (I.)	Fe_2O_3	4·80	305	0·025
13. ,, (II.)	,,	4·91	315	0·0368
14. Specular ,, (I.)	,,	4·89	222	0·205
15. ,, (II.)	,,	4·75	222	0·107
16. ,, (III.)	,,	4·90	113	0·517
17. Brown Hæmatite ...	$2Fe_2O_3 3H_2O$	3·70	315	0·0316
18. ,, powdered and heated	Fe_2O_3	—	315	0·0175
19. Chrome Iron Ore (I.)	$(Fe Mg) O . (Cr, Al)_2O_3$	3·88	315	0·0565
20. ,, (II.)	,,	4·15	315	0·079

† Maximum values of K_m found for cut bars, by ballistic method.

In view of the fact that nearly all the tests were made with powders, it was necessary to adopt the mass-susceptibility as a standard. In this respect, therefore, the present results differ from those given in the previous communications referred to, in which the volume-susceptibility was used.

It should be mentioned that, with the exception of natural Magnetite (1 and 2 of Table I.), all the determinations were made with a magnetic balance, either of the form described in the Proceedings of the Royal Society, A., Vol. 96, 1920, p. 429, or of the portable type described in the Proceedings of the Physical Society, Vol. XXXI., Pt. V., 1919, p. 338.

Natural Minerals.

The minerals comprised in Table I. can be conveniently divided into three groups. Nos. 1-7 and 19-20 are of the type represented by the formula $R''O . X'''_2O_3$. Nos. 8-11 are representatives of naturally crystallised alumina. Nos. 12-18 are examples of native ferric oxide in various forms.

As regards the first group, the first four contain ferric oxide combined with ferrous oxide or zinc oxide, whilst in the next three ferric oxide is replaced by alumina, the basic oxide being iron magnesium or zinc. As might be expected where the whole of the iron is substituted by elements like magnesium and aluminium, as in spinel (No. 7), the susceptibility falls to an almost insignificant figure. Where iron is present in the basic (FeO) or in the acidic (Fe_2O_3) condition, the susceptibility is higher in the latter case. For example, Franklinite (3, 4) gave susceptibilities $0{\cdot}455 \times 10^{-3}$ and $0{\cdot}64 \times 10^{-3}$, whereas Pleonaste (5) and Chrome Iron (19, 20) gave values so low as $0{\cdot}0243 \times 10^{-3}$, and about $0{\cdot}07 \times 10^{-3}$ respectively.

It might be thought from the importance of the ferric oxide in compound minerals as affecting susceptibility that this substance by itself would exhibit a high value. That this is not generally the case is shown by the low values given by red and brown Hæmatites (12, 13, 17). The specular ores (14, 15, 16) are all considerably higher, reaching the value of $0{\cdot}517 \times 10^{-3}$ (16). This shows that in native minerals the susceptibility of ferric oxide varies within wide limits, and this matter will be more fully dealt with later. In the case of these minerals in the solid it is noticeable that the higher susceptibility is associated with almost black crystalline form, and the red and brown Hæmatites are distinctly lower in susceptibility. When artificially prepared in the form of powders, considerable variations in colour are obtained, but give no indication as to the magnitude of the susceptibility. It is, of course, well known that the colour depends upon fineness of division and mode of preparation.

The close analogy between alumina and ferric oxide and the similarity of their mineral compounds induced the authors to examine the susceptibility of crystalline alumina in the form of Corundum, Sapphire and Ruby (8, 9, 10, 11), as well as the aluminates 5, 6, 7. The results show that the susceptibilities are very low, and, in the case of Corundum crystal (8), a small negative value has been found. The authors wish cordially to acknowledge the kindness of Dr. W. T. Gordon in supplying them with specially selected specimens.

IRON OXIDES AND FERRITES.

It is frequently stated that ferric oxide, in comparison with ferrous oxide, is practically a non-magnetic substance. The relatively high susceptibility of certain minerals in which there is no ferrous oxide suggests that this is not necessarily the case. For the purposes of comparison ferrous oxide and artificially-prepared magnetic oxide have been examined. The susceptibility of specimens of ferrous oxide, obtained by heating ferrous oxalate in the absence of air, gave a value of roughly one-half that obtained for artificially-prepared magnetic oxide. This result is given with some reserve, as it is impossible to guarantee the entire absence of magnetic oxide. The K_m value for precipitated magnetic oxide varied from 0·072 to 0·043 C.G.S. units, in a field $H=20{\cdot}4$, according to the method of preparation. As regards ferric oxide, the lowest value was obtained from powdered brown hæmatite, dehydrated by heating to bright redness: $17{\cdot}5\times10^{-6}$. In ascending order of magnitude the next example was obtained from basic ferric acetate by gradual heating, $K_m=63\times10^{-6}$. Ferric oxide, obtained from sodium ferrite by washing with hot water, gave the largely increased value 213×10^{-6}. This latter figure is greatly surpassed in the case of ferric oxide produced by the ignition of pyrophoric ferrous oxide. Rapid oxidation by full exposure to the air and subsequent heating on platinum foil gave a ferric oxide whose susceptibility was $4{\cdot}9\times10^{-3}$, and by slower oxidation the figure obtained was $28{\cdot}1\times10^{-3}$. A high susceptibility was exhibited by ferric oxide obtained by heating dried artificial magnetic oxide $K_m=26\times10^{-3}$. When the hydrated magnetic oxide was oxidised on platinum foil, a lower but still high value was obtained.

An important but neglected Paper "On Iron Rust Possessing Magnetic Properties," by Professor A. Liversidge, M.A., F.R.S.,* deals with this interesting subject, and he shows that ferric oxide which has been slowly produced on iron by long exposure to the air is comparatively a highly magnetic substance, even in cases in which chemical analysis showed a complete absence of ferrous oxide. In this Paper reference is made to earlier work on this subject, notably that of J. Robbins (*Chemical News*, Vol. I., 1859, p. 11), who obtained a peroxide of iron by igniting magnetic oxide, which still possessed strongly magnetic properties although proved to be free from ferrous oxide. Malaguti† described a magnetic variety of ferric oxide. A sample of iron rust was finely powdered and examined for ferrous oxide, no trace being found. Its susceptibility was $4{\cdot}4 \times 10^{-3}$, and this confirms the statement above referred to. After heating to redness on platinum foil the value of K_m was reduced to $3{\cdot}0 \times 10^{-3}$. C.G.S.

As a comparison with the oxide formed in the rusting of iron, an outer layer of chalybite, which by its colour was in a highly oxidised state, caused by weathering, was reduced to powder, which had a value of K_m equal to $34{\cdot}7 \times 10^{-6}$. The original chalybite in the deeper layers was then tested, and found to have a susceptibility of 103×10^{-6}. In the conversion of the carbonate into ferric oxide by the action of air and water at atmospheric temperature the susceptibility is reduced to a value almost identical with the values found in cases of brown and red hæmatites.

On heating the true chalybite in powder freely exposed to the air, the resulting oxide (free from ferrous compounds) was found to have a susceptibility of $13{\cdot}8 \times 10^{-3}$, which is indicative of the effect of the conversion of ferrous compounds at a high temperature. The specimen of chalybite, used as above, contained, as is usually the case, a small percentage of manganese and magnesium carbonates. In conformity with other experiments described in this paper it is possible that the high value of the susceptibility is in part due to the influence of the oxides of these metals.

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* Transactions of the Australasian Association. Hobart Meeting, 1892.

† "Ann. de Chim. et de Phys." Series (3) 69, 214.

It should be mentioned that the ferric oxides examined by the authors were chemically tested for ferrous oxide with a negative result.

Moissan* stated that magnetic oxide of iron exists in two modifications, one having a density of 4·86, obtained by reducing ferric oxide at 500° C., and the other a density of 5·18, obtained at higher temperatures, and Berthelot† showed that the form of lower density can be transformed into the one of higher density by heating strongly in nitrogen. He further drew a distinction between the two forms in that the one of lower density is soluble in nitric acid, and easily oxidised by heating in air on platinum foil, while the denser form resists both actions. The authors found that, although artificial magnetic oxide was easily converted into ferric oxide by heating in air, the finely powdered natural crystal (Traversella) resisted all attempts to oxidise it completely; even fusion with potassium nitrate produced only slight oxidation. In the case of magnetite from Altenfjord the oxidation was much more pronounced.

Early workers on this subject, notably Elliot (1862), Karl List‡ (1878), Percy, Ebelmen, Daubrée, Reich and others, obtained ferrites, such as franklinite, many of which were variously described as "feebly" or "strongly" magnetic. More recently Bemmelen and Klobbie§ and S. Hilpert‖ have examined a large number of ferrites with special reference to their transformation temperatures, but apparently no determinations of the actual susceptibilities were made.

Largely from this standpoint, it appeared desirable to prepare other ferrites, in which the ferrous oxide was substituted by some other oxide. Owing to the close similarity of cobalt to iron, one of the first to be tested was cobalt ferrite. The method employed in the preparation of this and other ferrites was the precipitation of the hydrated oxides from solutions in molecular proportions of the soluble salts of the metals concerned, either by sodium or ammonium hydroxide according to their relative suitability. The resulting precipitate, after washing, was collected and dried at from 95° to 100°C., and the susceptibility in the hydrated state was determined. The proportion of water actually found was not always that indicated by theory for a true hydroxide; nor is it possible to say how the water is shared between the oxides in a given case. The hydrates were rendered anhydrous by strongly heating on platinum foil, and the susceptibility of the resulting ferrite was then found. This method was the one employed by List, who obtained a number of ferrites, and mentions that many of them are magnetic. Hydrated cobaltous and ferric oxides had a susceptibility of 107×10^{-6}. After heating to about 900°C. the susceptibility was found to have increased about a hundred-fold. Similar results were obtained in the case of nickel.

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* "Ann. de Chimie" (5), 21, 233.

† "Ann de Chimie" (5), 23, 118.

‡ "Deut. Chem. Gas. Ber.," 1878, p. 1512.

§ "Journ. f. prakt. Chem." (2), 46, 496 (1892).

‖ "Deut. Chem. Ges. Ber." XLII. (1909), B.2., p. 2248.

Manganese presents a difficulty in the endeavour to form a ferrite, $MnO . Fe_2O_3$, as it appears possible that manganous oxide may reduce ferric oxide at a high temperature. A mixture of manganous and ferric sulphates, in molecular proportions, was precipitated by sodium hydroxide, and the precipitate washed and dried with the least possible air exposure, as manganous hydroxide is well known to absorb oxygen readily. Before heating the mixture gave a value of 98×10^{-6} for K_m, and after ignition on platinum foil it had increased to $26{,}900 \times 10^{-6}$, but, as this enormous increase may in part be due to the formation of true magnetite, the result must be accepted with some reservation. The following equation might express the change: $3\,(MnO . Fe_2O_3) = Mn_3O_4 + 2Fe_3O_4$. Chemical analysis of the product would not show the distribution of oxygen, as on solution in an acid, *e.g.*, hydrochloric, the higher oxide of manganese, would convert the ferrous into a ferric salt, and no ferrous compound would be indicated, although it might have existed as such in the solid.

Remarkable figures were obtained with copper ferrite. Precipitated from hot solutions, but not boiling, the susceptibility was 102×10^{-6}. From boiling solutions the value was found to be 486×10^{-6}. Both of these after ignition gave the very high figure $33{,}000 \times 10^{-6}$. The susceptibility of copper ferrite therefore considerably exceeds cobalt or nickel ferrites, or, indeed, any similarly constituted compound excepting magnetite itself. Hilpert (*loc. cit.*) has found that a lead-copper ferrite, if rapidly cooled from 900 to 1,000°C., is as strongly magnetic as copper ferrite, but if slowly cooled (five to six hours) it is weakly magnetic. The authors have examined copper ferrite in this connection, and have found that slow cooling (four hours) from about 960°C. reduced the susceptibility to about one half. Subsequent heating to the same temperature, followed by rapid cooling, restored the susceptibility to nearly its original value. In this connection it is interesting to note that copper pyrites shows a considerable increase in susceptibility on being heated in the presence of air with the formation of the double oxides. The susceptibility of one specimen of powdered $CuFeS_2$ increased from $6{\cdot}4\times10^{-6}$ (its initial value) to $4{,}480\times10^{-6}$.

Zinc ferrite is interesting as being an artificial representation of franklinite, and, whereas the susceptibility was 77×10^{-6} before heat treatment, it was 258×10^{-6} after treatment. This final value is about a half of that given by the specimens of franklinite actually tested (*see* Table I.). Cadmium behaves similarly.

The increase is more marked in the case of magnesium, the actual figures being 79×10^{-6} and $3{,}100\times10^{-6}$ before and after heating. Calcium ferrite, prepared by strongly heating a mixture of slaked lime ($Ca(OH)_2$) and ferric oxide in a crucible, was found to have a susceptibility of $8{\cdot}1\times10^{-3}$. Subsequent boiling with water and drying in air and re-heating reduced this figure to 3×10^{-3}.

Silver was chosen because on heating its oxide is resolved into the metal and oxygen, and consequently no ferrite, but a mixture of ferric oxide and metallic silver would result. As in the other cases, it showed an increase in susceptibility, the figures being $93{\cdot}6\times10^{-6}$ and $6{,}700\times10^{-6}$. Metallic silver was finally removed by treatment with hot nitric acid, and the susceptibility of the resulting ferric oxide was found to be $8{,}380\times10^{-6}$—a not unexpected result if the silver merely acted as an inert body. Evidently during the change the ferric oxide in contact with the silver oxide had acquired a marked increase in susceptibility, which it retained after the reduction of the silver oxide and removal of the silver itself.

The figures obtained in the case of mercury ($K_m=65{\cdot}8\times10^{-6}$ and 915×10^{-6}), though not so marked as in the case of silver, show that a similar effect is produced when the mercury is entirely volatilised in the process of heating.

Up to this point basic oxides have been employed in conjunction with ferric oxide. A variation was now made by using aluminia and ferric oxide, the susceptibility being $25{\cdot}6\times10^{-6}$ before, and $1{,}080\times10^{-6}$ after heat treatment. Chromic and ferric oxides show a similar result, though less marked ($K_m=40{\cdot}5$ and 418×10^{-6}). The result in the last two examples indicates that a definitely basic oxide is not essential to the production of enhanced susceptibility. It appears, therefore, that whenever ferric oxide is heated in intimate contact with other oxides, more especially the basic ones, *e.g.*, lime, combination may occur leading to a considerable increase in the susceptibility.

ALUMINA AND ARTIFICIAL ALUMINATES.

In Table I. it is shown that corundum crystal (8) has a small negative susceptibility. The hydrated alumina has also a negative susceptibility, almost of identical value $-0{\cdot}337 \times 10^{-6}$. After ignition the susceptibility was still negative, but smaller in value—$0{\cdot}098 \times 10^{-6}$. Special care had to be taken in the preparation of this substance, as the presence of a minute trace of iron would cause its susceptibility to become positive in value. Additional figures were obtained from varieties of aluminates. Cobalt (38 to 43×10^{-6}) and copper ($6{\cdot}2$ to $9{\cdot}2 \times 10^{-6}$) show a small increase after heat treatment, and manganese a small diminution (76 to $71{\cdot}2 \times 10^{-6}$). The result in the case of copper is specially

TABLE II.

Certain Hydroxides and Oxides.

Substance and Formula.	H in C.G.S. units.	K_m in 10^{-6} C.G.S. units.
Nickel hydroxide $Ni(OH)_2$	315	48·3
Nickel Oxide NiO	305	53·7
Cobaltous oxide CoO	318	74·5
Cobalto-cobaltic oxide Co_3O_4	318	39 to 43·6
Cobaltic oxide Co_2O_3	318	34·3
Manganous oxide MnO	315	81·0
"Red oxide" Mn_3O_4	430	68·0
Manganic oxide Mn_2O_3	318	67·0
Manganese dioxide MnO_2	288	66·2
Chromic hydroxide $Cr_2(OH)_6 . H_2O$	300	45·5
Chromic oxide Cr_2O_3	300	29·4
Cuprous oxide Cu_2O	670	1·2
Hydrated cupric oxide $Cu(OH)_2 . CuO$	305	5·0
Cupric oxide CuO	305	3·8
Zinc hydroxide $Zn(OH)_2$	724	—0·487
Zinc oxide ZnO	717	—0·305
Hydrated alumina $Al_2O_32H_2O$	650	—0·337
Alumina Al_2O_3	670	—0·098

interesting, as it shows the vast difference in behaviour between a copper aluminate and a copper ferrite. Copper chromite, $CuO.Cr_2O_3$, also has a low susceptibility, $26{\cdot}7 \times 10^{-6}$.

CERTAIN HYDROXIDES AND OXIDES.

In the course of the investigation a number of pure oxides had been prepared and their susceptibilities measured, and these are given in Table II. It is interesting to note that, in the case of cobalt and manganese, the susceptibilities of the different oxides show a steady decrease with increase of oxygen, and, unlike iron, there is no increase in the sus eptibility associated with Co_3O_4 or Mn_3O_4 corresponding with Fe_3O_4.

The susceptibility of pure copper is well known to be negative, but its oxides are all positive, though small in magnitude.

The figures obtained in the cases of chromium and zinc call for no special comment, but are given because these substances have been employed in the earlier stages of the work. It is mentioned in Watts' "Dictionary of Chemistry" that Wohler many years ago described a magnetic oxide of chromium. Recently Soné and Ishiwara * have investigated this substance, to which they assign the formula Cr_5O_9.

For further information on the subject of pure oxides standard books of reference, such as Landolt and Börnstein, should be consulted, where the results obtained by various investigators will be found. It is noteworthy, however, that in the 1912 edition of the above work no values are given for the oxides of iron, cobalt or nickel and the results quoted for other oxides show considerable variation as measured by different workers.

DISCUSSION.

Prof. BRAGG showed a model of the molecule of magnetite as deduced from its X-ray spectra, and pointed out various conclusions which could be drawn from it. For instance, it was impossible to deduce the properties of the magnetite from those of Fe_2O_3 and FeO, nor could any of its properties be ascribed to one or the other, because the crystalline form of the Fe_2O_3 and FeO in the magnetite molecule differed from their crystalline form when separate. It was possible by X-ray spectroscopy to say what crystals were present in any powder. No doubt magnetic effects were associated with the crystal structure; but experiments on pyrrotine showed no difference between the magnetised and unmagnetised state.

Dr. RAYNER asked if differences in aggregate form rather than in crysta form might not account for the different magnetic properties of some substances of the same composition.

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* "Science Abstracts," Vol. XVIII., No. 926.

Prof. **Wilson** said he was disappointed by Prof. Bragg's remark about pyrrotine as it partially answered the question he wished to ask. Would the X-ray spectroscope be likely to show any difference in the two ferric oxides of such widely different susceptibilities mentioned in the Paper ?

Prof. **Bragg** : If there is any difference in crystal structure it would show it.

Mr. **Guild** said the case of pyrrotine was different from that of the two ferric oxides. The experiment with pyrrotine showed that there was no difference in the molecular structure of a magnetisable substance when magnetised and unmagnetised. This one would expect since the phenomena of magnetisation are accounted for by the orientation of the molecules, and not by any alteration of their properties. The case of the two ferric-oxides in which Prof. Wilson was interested was quite different. Here we had magnetisable and non-magnetisable modifications of the substance. In this case one would expect to find a difference in molecular structure revealed by the X-ray spectrometer.

Dr. J. S. G. **Thomas** asked if the differences might not be due to differences in the orientation of the magnetic couplings between the electronic orbits. Did X-ray analysis afford any evidence of such differences of orientation ?

Prof. **Bragg** : That is the problem we are all working at. In the case of the diamond there is no doubt that the carbon atoms point in opposite directions.

Prof. **Herroun** said that in pyrrotine the susceptibility in one direction was 70 times that in another. Would the X-ray spectra give any clue to this ?

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